

Evaluating uncertainty

Monte Carlo evaluation
in a spreadsheet

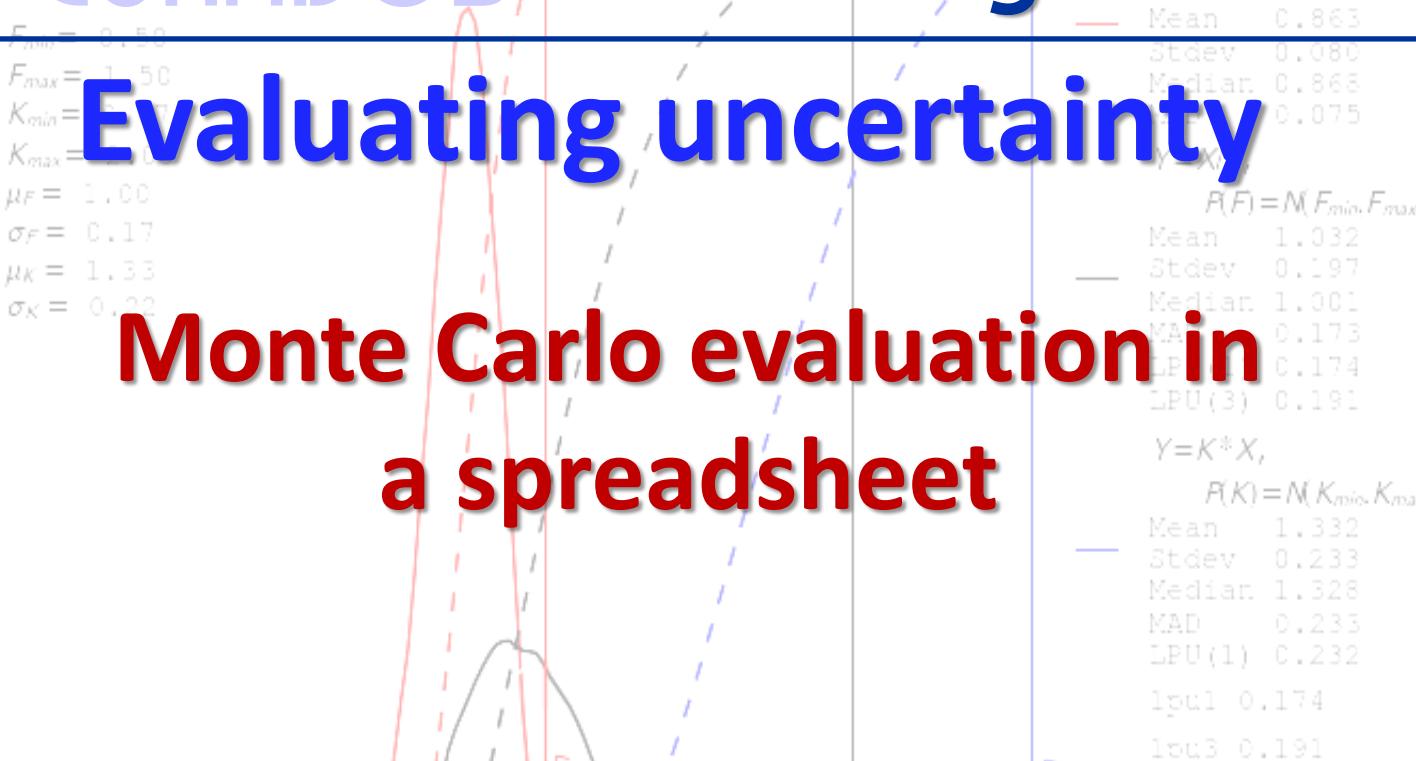
Janwillem van Dijk

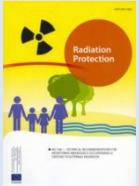
Eurados Training Course
Lisbon, Portugal, 18-22 May 2015



$F_{min} = 0.750$
 $F_{max} = 1.50$
 $K_{min} = 0.750$
 $K_{max} = 1.50$
 $\mu_F = 1.00$
 $\sigma_F = 0.17$
 $\mu_K = 1.33$
 $\sigma_K = 0.22$

dose





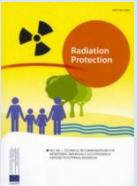
mcm_example.xlsx, mcm_example.ods

$$x = f_R f_{\text{Det}} (f_{E,\alpha} H_{\text{True}} + f_{\text{Bg}} h_{\text{Bg}}) + z$$

$$y = \frac{x - Z}{F_R F_{E,\alpha} F_{\text{Det}}} - \frac{F_{\text{Bg}}}{F_{E,\alpha}} Y_{\text{Bg}}$$

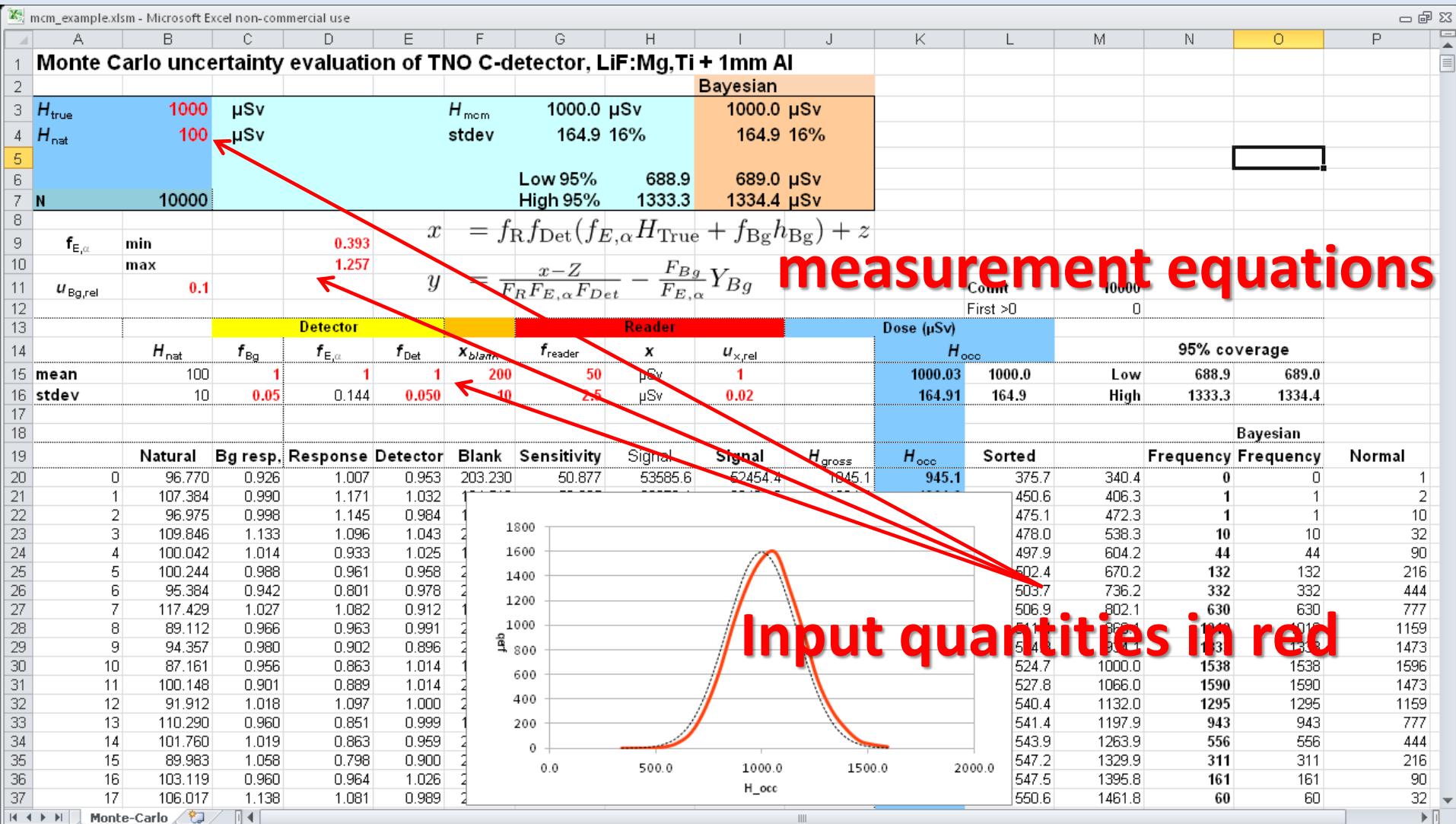
x: the signal you will see from the reader
y: the dose you will report

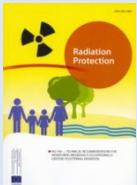
Symbols in capital are expectations



Evaluating uncertainty

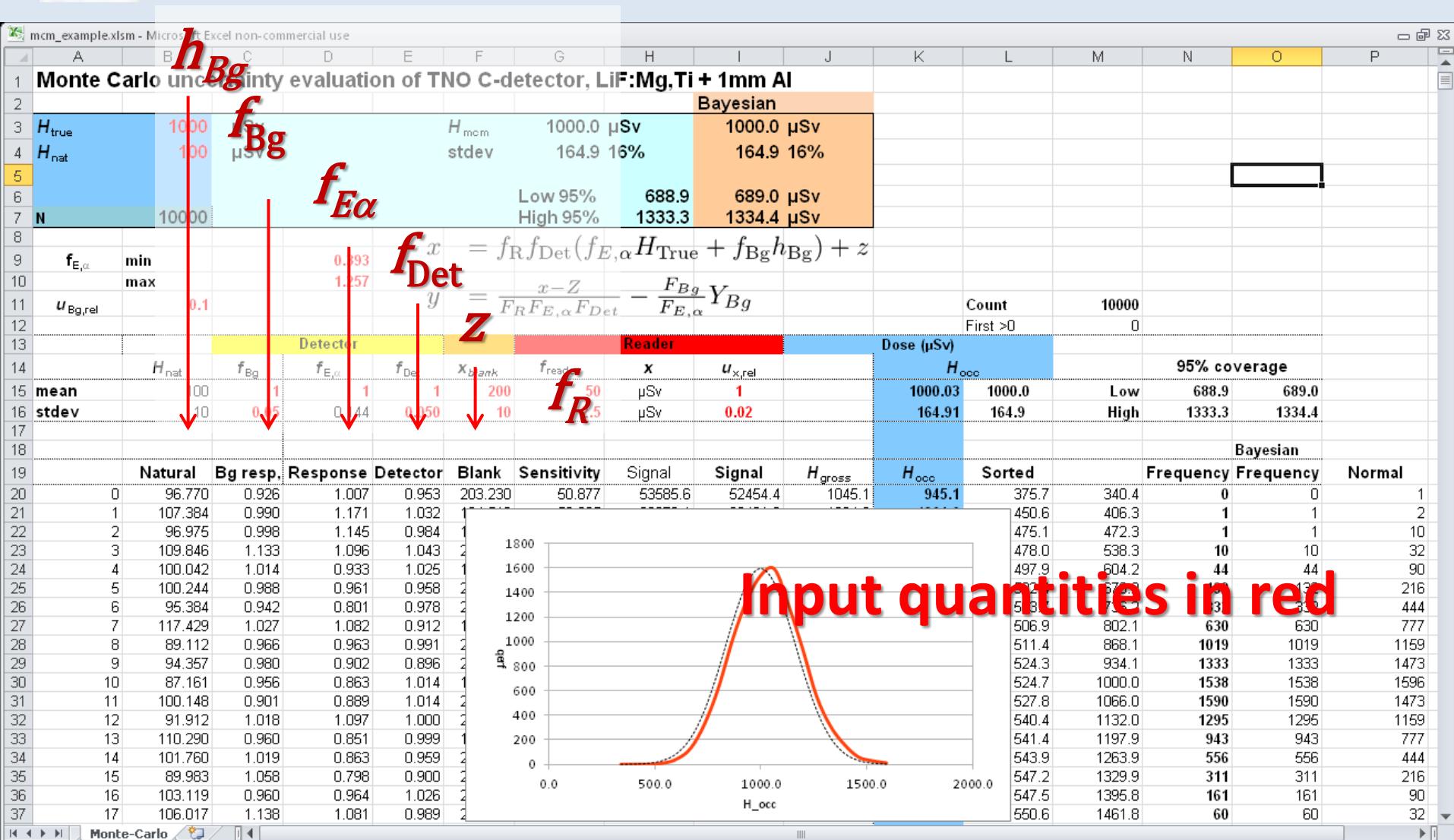
EURADOS

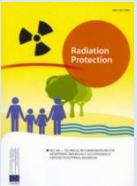




Evaluating uncertainty

EURADOS



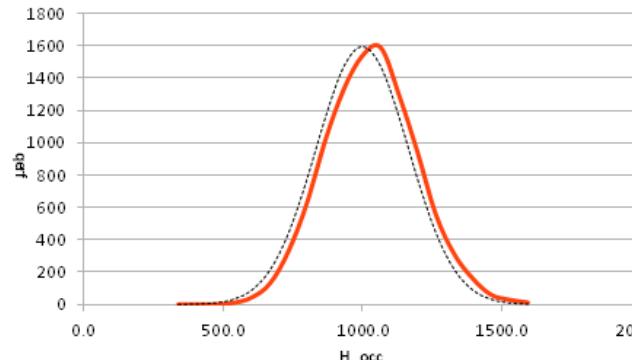


Evaluating uncertainty

EURADOS

The signal from the reader
Gross dose
Net dose

Monte Carlo uncertainty evaluation of TNO C-detector, LiF:Mg,Ti + 1mm Al									
H_{true}	1000	μSv	H_{mcm}	1000.0	μSv	Bayesian			
H_{nat}	100	μSv	stdev	164.9	16%	1000.0 μSv	164.9 μSv	16% 16%	
N	10000		Low 95%	688.9	689.0 μSv				
			High 95%	1333.3	1334.4 μSv				
$f_{E,\alpha}$	min	0.393	$x = f_R f_{Det}(f_{E,\alpha} H_{True} + f_{Bg} h_{Bg}) + z$						
	max	1.257	$y = \frac{x - Z}{F_R f_{E,\alpha} F_{Det}} - \frac{F_{Bg}}{f_{E,\alpha}} Y_{Bg}$						
$u_{Bg,rel}$	0.1								
	Detector			Reader			Dose (μSv)		
	H_{nat}	f_{Bg}	$f_{E,\alpha}$	f_{Det}	x_{blank}	f_{reader}	x	$u_{x,rel}$	H_{occ}
mean	100	1	1	1	200	50	μSv	1	1000.03
stdev	10	0.05	0.144	0.050	10	2.5	μSv	0.02	164.91
	Natural	Bg resp.	Response	Detector	Blank	Sensitivity	Signal	Signal	H_{gross}
	0	96.770	0.926	1.007	0.953	203.230	50.877	53585.6	52454.4
	1	107.384	0.990	1.171	1.032	1	1800		1045.1
	2	96.975	0.998	1.145	0.984	1	1600		945.1
	3	109.846	1.133	1.096	1.043	2	1400		375.7
	4	100.042	1.014	0.933	1.025	1	1200		340.4
	5	100.244	0.988	0.961	0.958	2	1000		0
	6	95.384	0.942	0.801	0.978	2	800		0
	7	117.429	1.027	1.082	0.912	1	600		1
	8	89.112	0.966	0.963	0.991	2	400		450.6
	9	94.357	0.980	0.902	0.896	2	200		406.3
	10	87.161	0.956	0.863	1.014	1	0		475.1
	11	100.148	0.901	0.889	1.014	2			472.3
	12	91.912	1.018	1.097	1.000	2			478.0
	13	110.290	0.960	0.851	0.999	1			497.9
	14	101.760	1.019	0.863	0.959	2			502.4
	15	89.983	1.058	0.798	0.900	2			503.7
	16	103.119	0.960	0.964	1.026	2			506.9
	17	106.017	1.138	1.081	0.989	2			511.4



Evaluating uncertainty

mcm_example.xlsx - Microsoft Excel non-commercial use

Monte Carlo uncertainty evaluation of TNO C-detector, LiF.Mg,Ti, 1mm Al

Bayesian

Statistics

Data sorted with macro

Statistics at end of column

Detector Reader Dose (µSv)

95% coverage

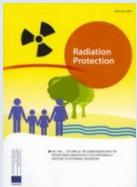
Bayesian

Normal distribution

Monte-Carlo

RP160 Training course Lisbon 2015

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	
1	Monte Carlo uncertainty evaluation of TNO C-detector, LiF.Mg,Ti, 1mm Al																
2																	
3	H_{true}	1000	µSv		H_{mcm}	1000.0	µSv		1000.0	µSv							
4	H_{nat}	100	µSv		stdev	164.9	16%		164.9	16%							
5																	
6									Low 95%	688.9							
7	N	10000							689.0	µSv							
8									High 95%	1333.3							
9	$f_{E,\alpha}$	min															
10																	
11		max															
12	$u_{Bg,rel}$	0.1															
13		Detector				Reader				Dose (µSv)				95% coverage			
14		H_{nat}	f_{Bg}	$f_{E,\alpha}$	f_{Det}	x_{blank}	f_{reader}	x	$u_{x,rel}$	H_{occ}	1000.03	1000.0	Low	688.9	689.0		
15	mean	100	1	1	1	200	50	µSv	1	1000.03	1000.0	High	1333.3	1334.4			
16	stdev	10	0.05	0.144	0.050	10	2.5	µSv	0.02	164.91	164.9						
17																	
18																	
19		Natural	Bg resp.	Response	Detector	Blank	Sensitivity	Signal	Signal	H_{gross}	H_{occ}	Sorted	Frequency	Frequency	Normal		
20	0	96.770	0.926	1.007	0.953	203.230	50.877	53585.6	52454.4	1045.1	945.1	375.7	340.4	0	1		
21	1	107.384	0.990	1.171	1.032	1						450.6	406.3	1	2		
22	2	96.975	0.998	1.145	0.984	1						475.1	472.3	1	10		
23	3	109.846	1.133	1.096	1.043	2	1800					478.0	538.3	10	32		
24	4	100.042	1.014	0.933	1.025	1						497.9	604.2	44	90		
25	5	100.244	0.988	0.961	0.958	2						502.4	670.2	132	216		
26	6	95.384	0.942	0.801	0.978	2						503.7	736.2	332	444		
27	7	117.429	1.027	1.082	0.912	1						506.9	802.1	630	777		
28	8	89.112	0.966	0.963	0.991	2						511.4	868.1	1019	1159		
29	9	94.353	0.998	0.998	0.998	1						524.3	934.1	1333	1473		
30	10	87.161	0.956	0.863	1.014	1						524.7	1000.0	1538	1596		
31	11	100.148	0.901	0.889	1.014	2						527.8	1066.0	1590	1473		
32	12	91.912	1.018	1.097	1.000	2						540.4	1132.0	1295	1159		
33	13	110.290	0.960	0.851	0.999	1						541.4	1197.9	943	777		
34	14	101.760	1.019	0.863	0.959	2						543.9	1263.9	556	444		
35	15	89.983	1.058	0.798	0.900	2						547.2	1329.9	311	216		
36	16	103.119	0.960	0.964	1.026	2						547.5	1395.8	161	90		
37	17	106.017	1.138	1.081	0.989	2						550.6	1461.8	60	32		

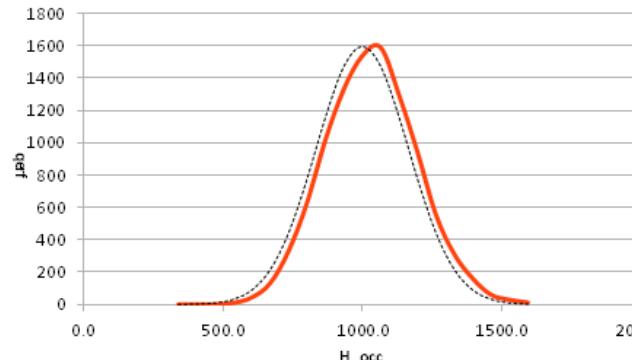


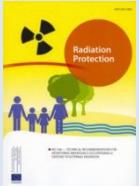
Evaluating uncertainty

EURADOS

Monte Carlo uncertainty evaluation of TNO C-detector, LiF:Mg,Ti + 1mm Al										
H_{true}	1000	μSv	H_{mcm}	1000.0	μSv	Bayesian				
H_{nat}	100	μSv	stdev	164.9	16%	1000.0	μSv	164.9	16%	
N	10000		Low 95%	688.9		689.0	μSv			
			High 95%	1333.3		1334.4	μSv			
$f_{E,\alpha}$	min	0.393	$x = f_R f_{Det}(f_{E,\alpha} H_{True} + f_{Bg} h_{Bg}) + z$							
	max	1.257	$y = \frac{x-Z}{F_R f_{E,\alpha} F_{Det}} - \frac{F_{Bg}}{f_{E,\alpha}} Y_{Bg}$			Count	10000			
						First >0	0			
	Detector				Reader			Dose (μSv)		
	H_{nat}	f_{Bg}	$f_{E,\alpha}$	f_{Det}	x_{blank}	f_{reader}	x	$u_{x,rel}$	H_{occ}	95% coverage
mean	100	1	1	1	200	50	μSv	1	1000.03	1000.0 Low 688.9 689.0
stdev	10	0.05	0.144	0.050	10	2.5	μSv	0.02	164.91	164.9 High 1333.3 1334.4
	Natural	Bg resp.	Response	Detector	Blank	Sensitivity	Signal	Signal	H_{gross}	Bayesian
	0	96.770	0.926	1.007	0.953	203.230	50.877	53585.6	52454.4	H_{occ}
	1	107.384	0.990	1.171	1.032	1				Sorted
	2	96.975	0.998	1.145	0.984	1				Frequency
	3	109.846	1.133	1.096	1.043	2	1800			Frequency
	4	100.042	1.014	0.933	1.025	1				Normal
	5	100.244	0.988	0.961	0.958	2				
	6	95.384	0.942	0.801	0.978	2				
	7	117.429	1.027	1.082	0.912	1				
	8	89.112	0.966	0.963	0.991	2				
	9	94.357	0.980	0.902	0.896	2				
	10	87.161	0.956	0.863	1.014	1				
	11	100.148	0.901	0.889	1.014	2				
	12	91.912	1.018	1.097	1.000	2				
	13	110.290	0.960	0.851	0.999	1				
	14	101.760	1.019	0.863	0.959	2				
	15	89.983	1.058	0.798	0.900	2				
	16	103.119	0.960	0.964	1.026	2				
	17	106.017	1.138	1.081	0.989	2				

Bayesian statistics





MCM number of samples needed

The sheet works with 10,000 samples

Recalculation (Press F9) will show different results

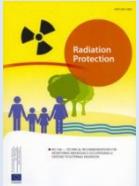
The figure changes shape

10,000 samples is in general far to few

IEC TR 62461:2015 recommends 1,000,000

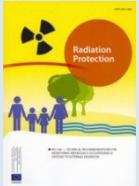
The JCGM gives an adaptive procedure

van Dijk, J.W.E. *Measurement models for passive dosimeters in view of uncertainty evaluation using Monte Carlo method*, Radiat. Prot. Dosim. 162(4), 438-445 (2014)



MCM number of samples needed

**My experience for the current type of problems:
300,000-500,000 will do
except where the measurand distribution is
very skewed**



Characteristic limits

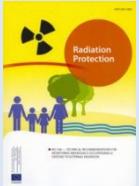
Decision threshold

The value above which you can decide that a dose above background level is observed

Detection limit

The smallest true value of the dose that can be detected with the dosimeter

ISO 11929:2010, *Determination of the detection limit and decision threshold for ionizing radiation measurements*



Characteristic limits

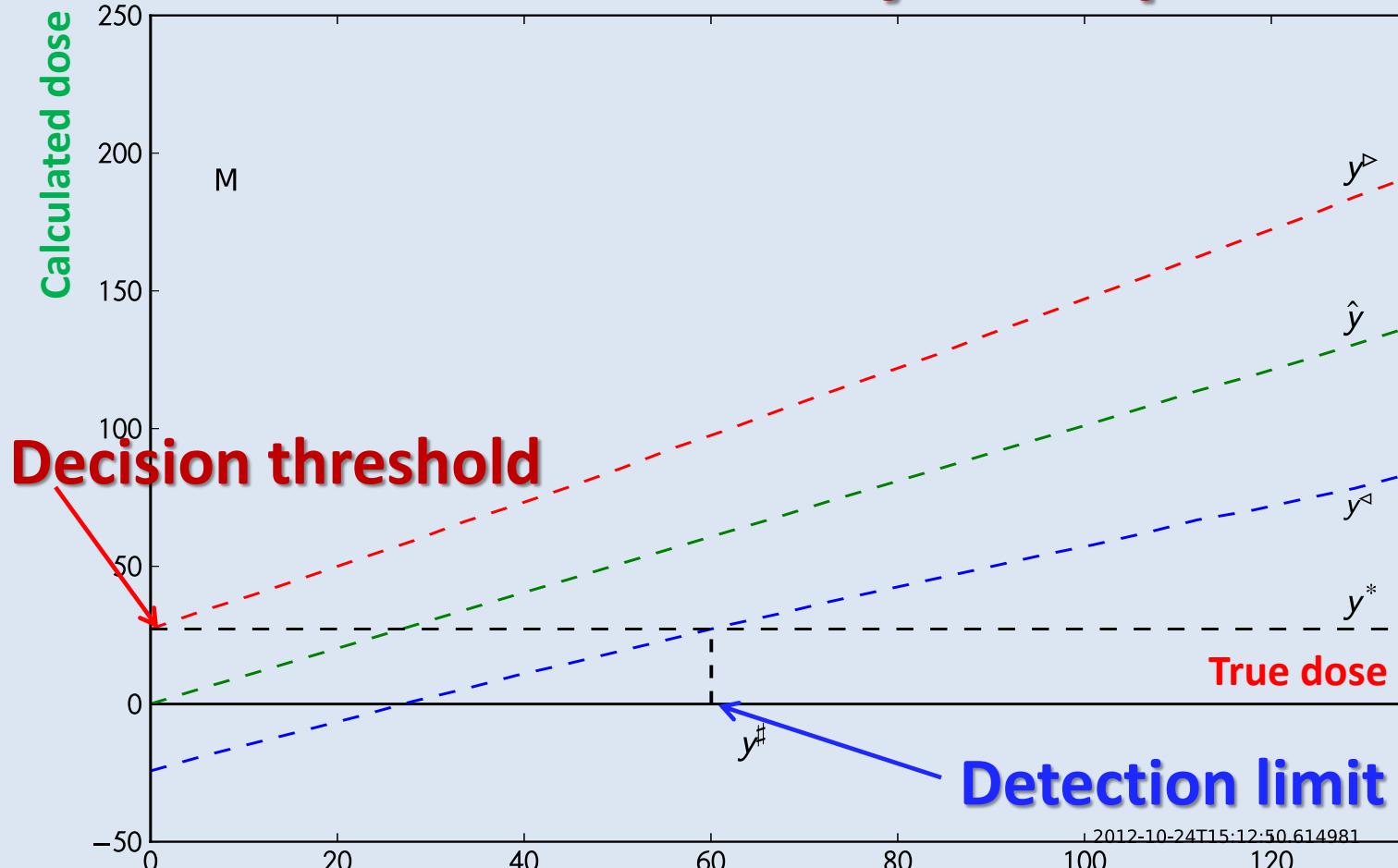
Rule of thumb:

$$y^* \quad \simeq 1.7u_y \mid y = 0.0$$

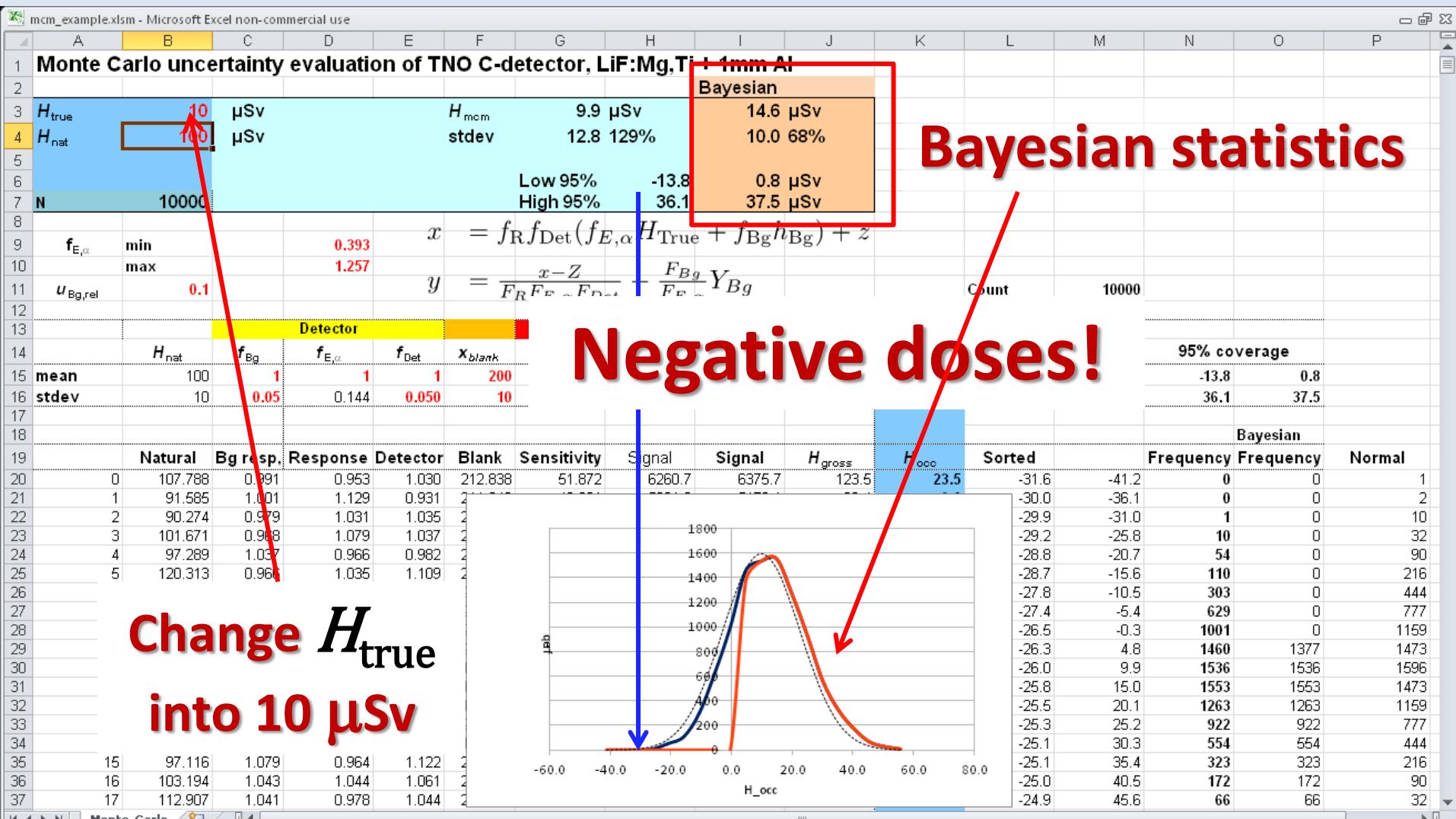
$$y^\# \quad \simeq 3.3u_y \mid y = 0.0$$

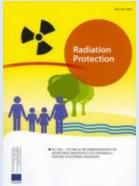
L.A. Curry, *Uncertainties in measurements close to the detection limit: Detection and quantification capabilities in IAEA Tecdoc 1401 (9-34)*

Characteristic limits by interpolation



Evaluating uncertainty





MCM and Bayes

Using Mont Carlo Bayesian statistics for

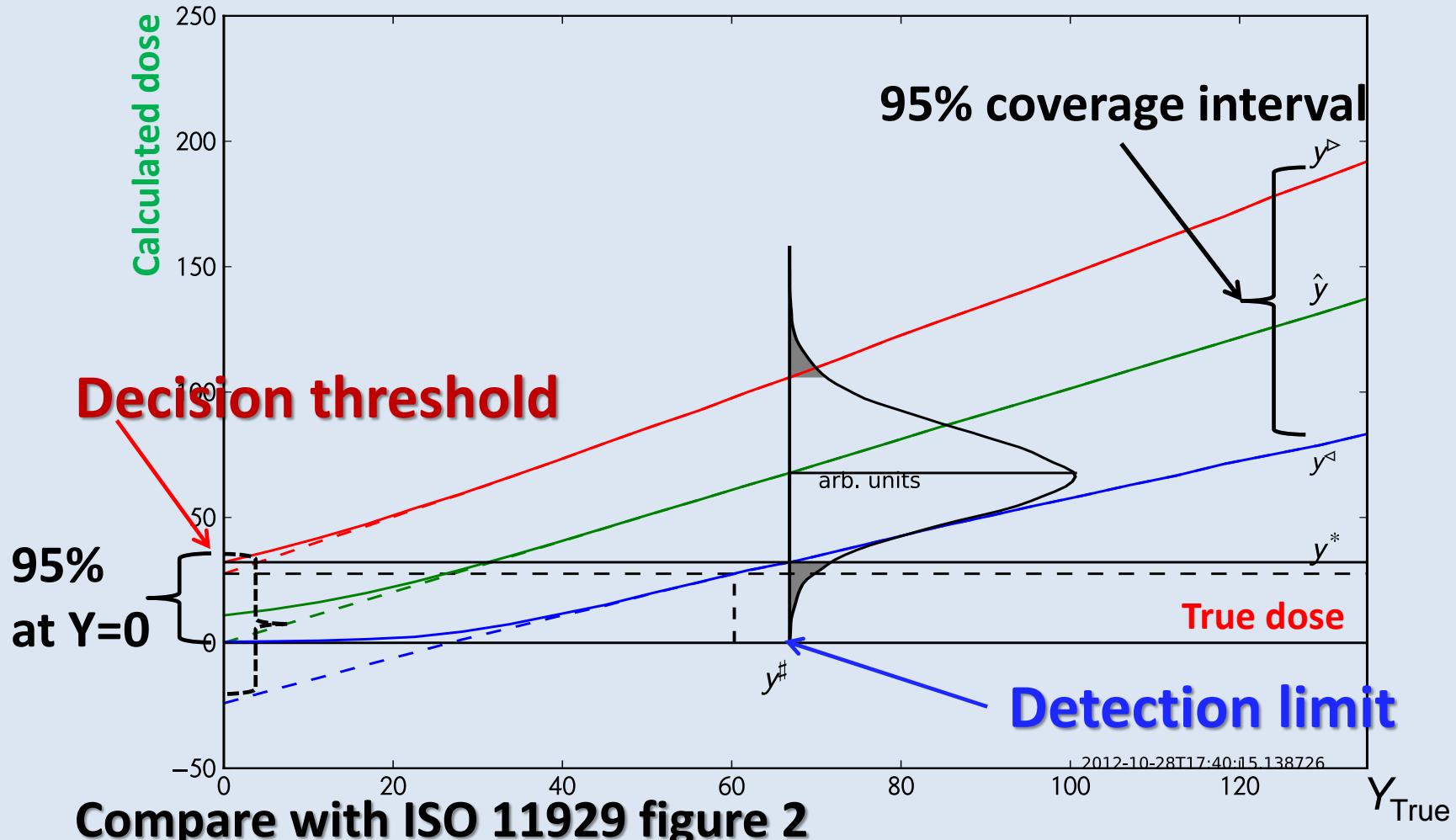
prior:

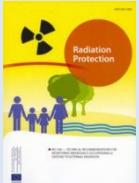
$$P(y < 0) = 0$$

is achieved by rejecting all negative samples

C. Elster, *Calculation of uncertainty in the presence of prior knowledge. Metrologia* 44 111-116 (2007)

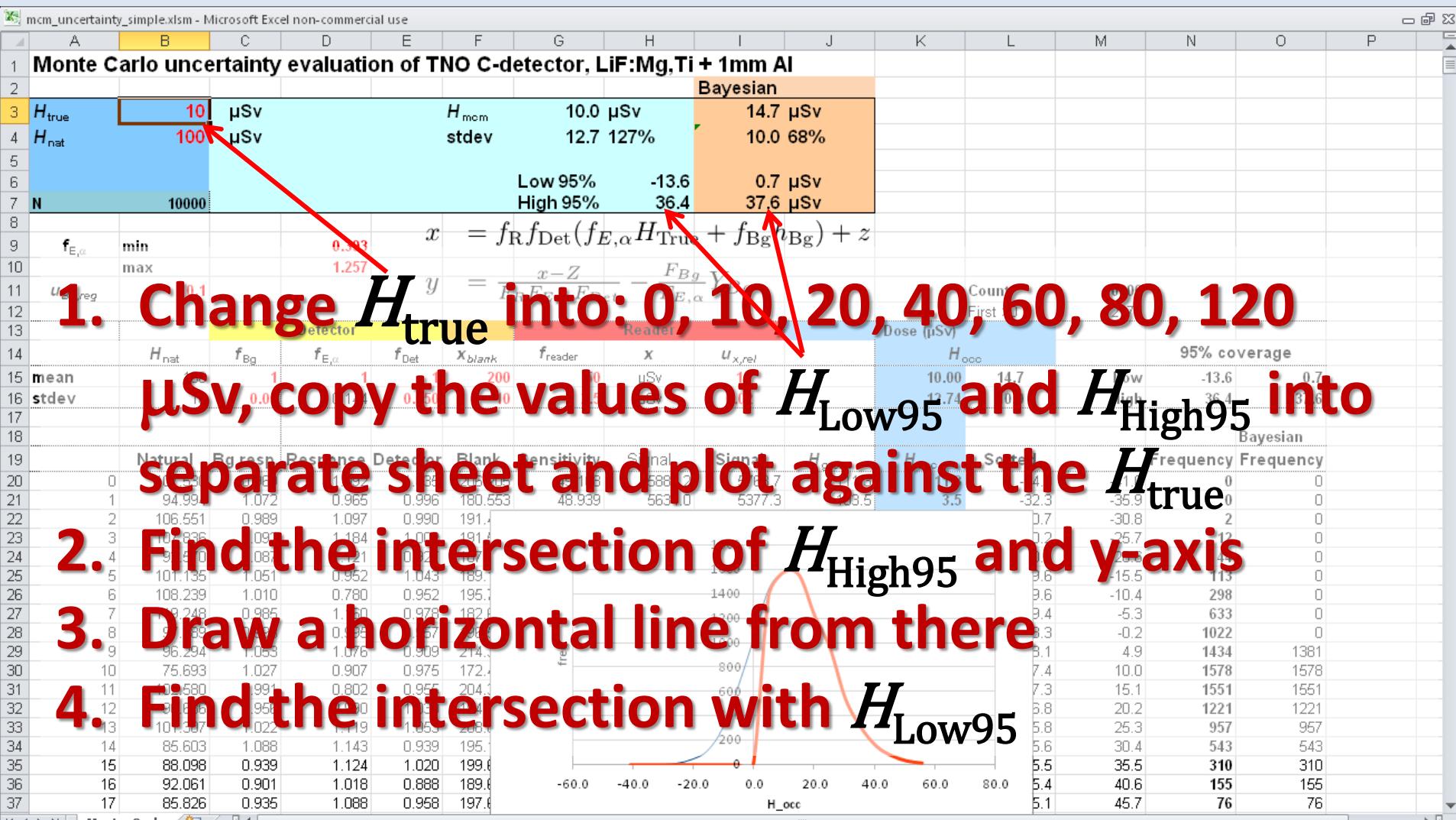
Characteristic limits by interpolation



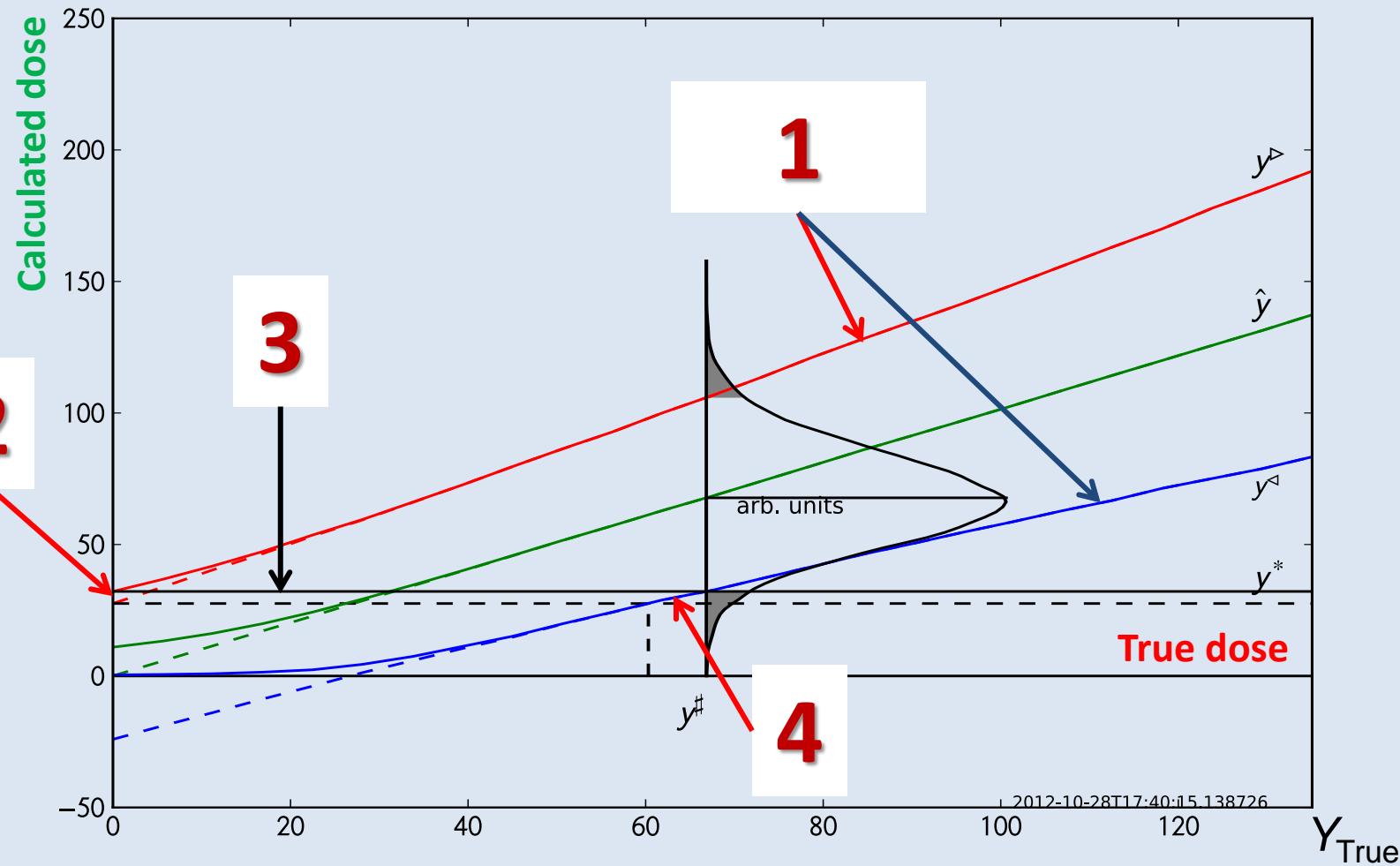


Evaluating uncertainty

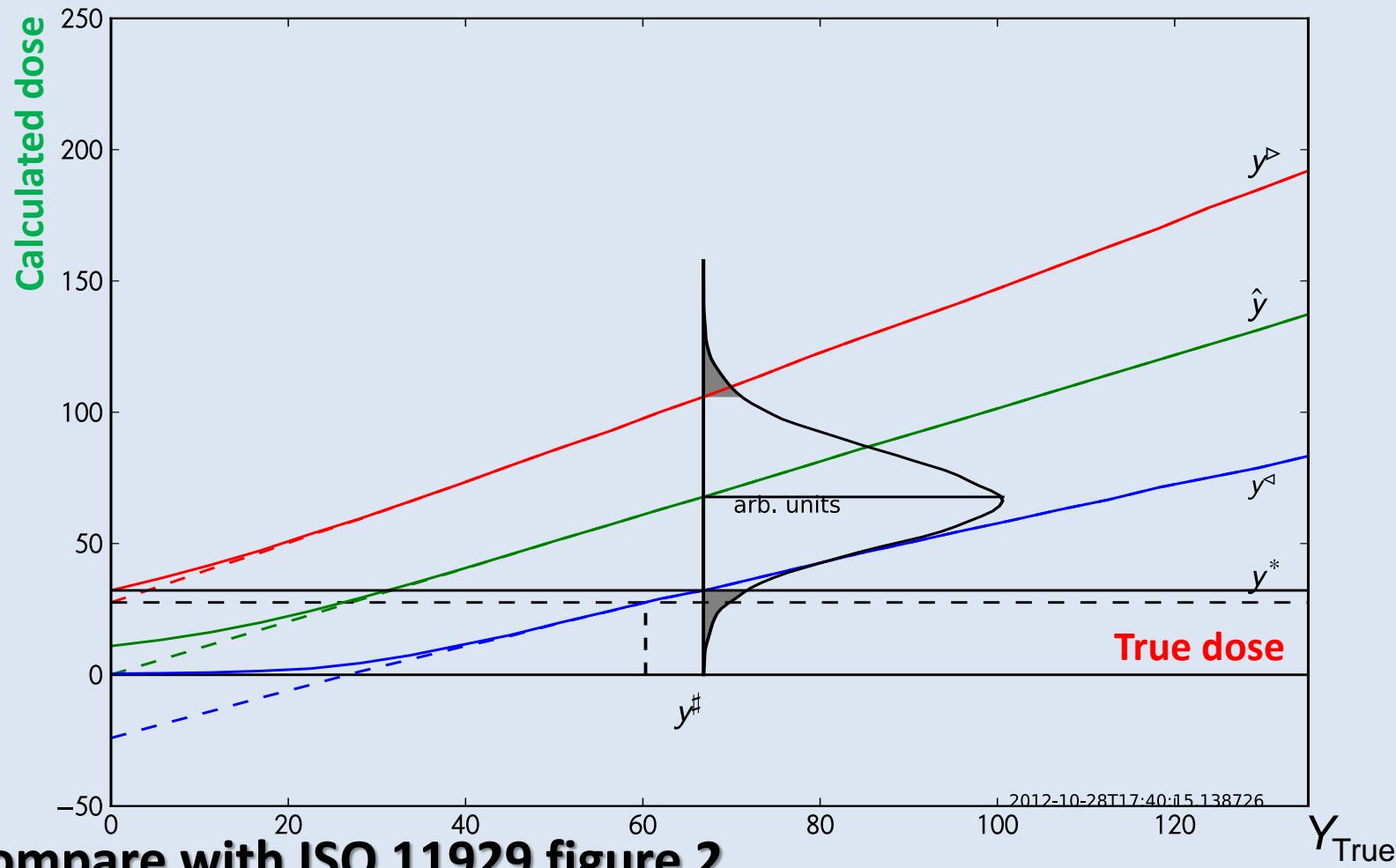
EURADOS



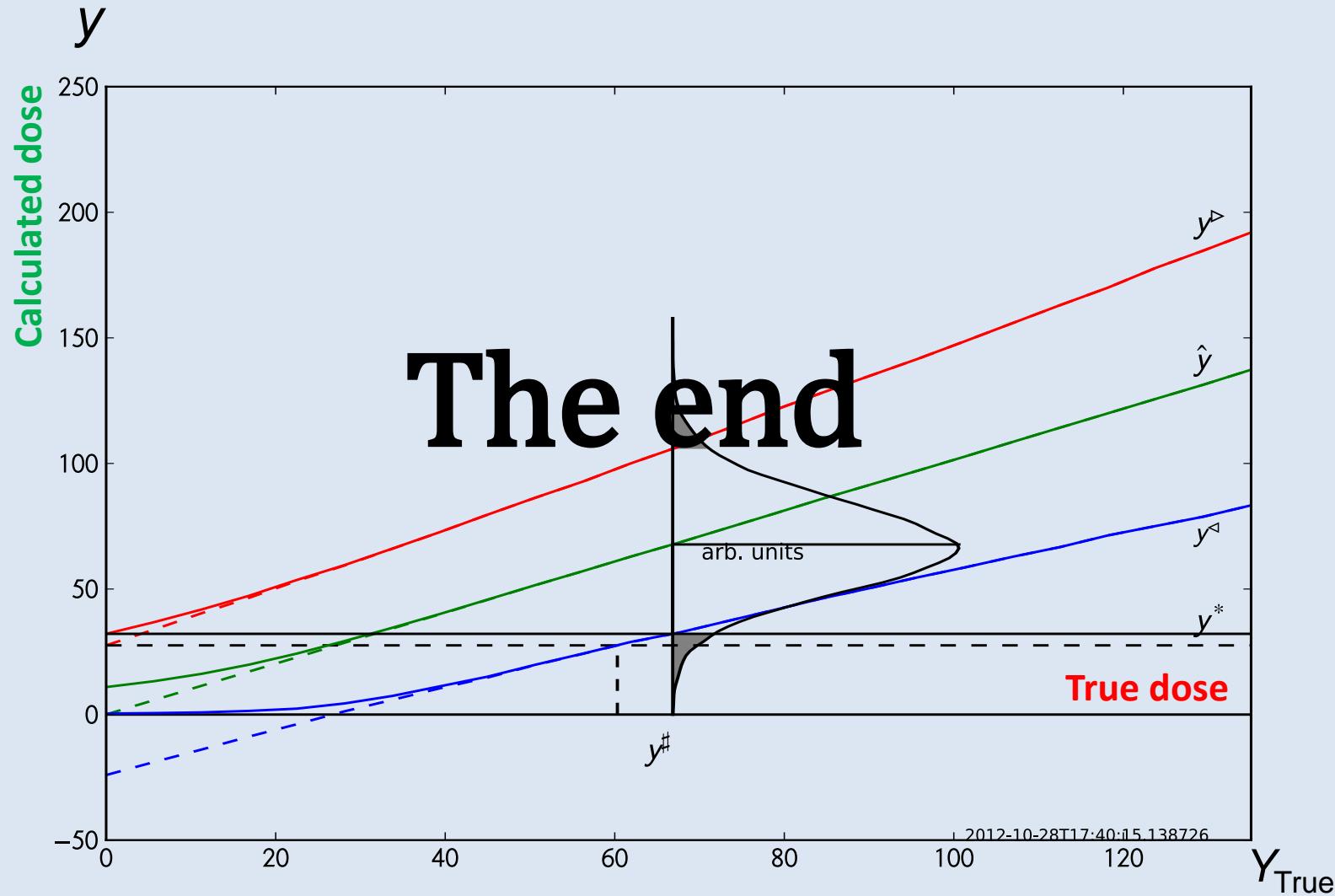
Characteristic limits by interpolation

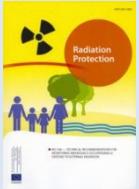


Characteristic limits by interpolation



Evaluating uncertainty





Important guiding documents

Joint Committee on Guides in Metrology

Guide to the expression of uncertainties in measurement and supplements
<http://www.bipm.org/en/publications/guides/gum.html> *)

European Commission

Technical Recommendations for Monitoring Individuals Occupationally Exposed to Ionizing Radiation
<http://ec.europa.eu/energy/sites/ener/files/documents/160.pdf> *)

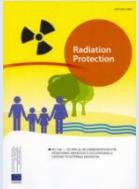
ICRU Report 76

Measurement quality assurance for ionizing radiation dosimetry
<http://www.icru.org>

IEC TR 62461

Radiation Protection Instrumentation – Determination of uncertainty
http://webstore.iec.ch/preview/info_iec62461%7Bed2.0%7Den.pdf

*) Document can be downloaded for free



Some Monte Carlo literature

J. W. E. van Dijk, *Measurement models for passive dosimeters in view of uncertainty evaluation using the Monte Carlo method.* Radiat Prot Dosimetry (2014) 162 (4): 438-445

H. Stadtmann and C. Hranitzky, *Uncertainty assessment of a two element LiF:Mg,Ti TL personal dosimeter using Monte-Carlo techniques.* Radiat Prot Dosimetry (2011) 144 (1-4): 67-71

R. Behrens, *Uncertainties in external dosimetry: analytical vs. Monte Carlo method.* Radiat Prot Dosimetry (2010) 138 (4): 346-352

J. W. E. van Dijk, *Developments in uncertainty analysis for individual monitoring.* Radiat Prot Dosimetry (2011) 144 (1-4): 56-61

J. W. E. van Dijk, *Evaluating the uncertainty in measurement of occupational exposure with personal dosimeters.* Radiat Prot Dosimetry (2007) 125 (1-4): 387-394

J. W. E. van Dijk, *Uncertainties in personal dosimetry for external radiation: a Monte Carlo approach.* Radiat Prot Dosimetry (2006) 121 (1): 31-38

Maurice Cox, Peter Harris, Gyeonghee Nam, and David Thomas, *The use of a Monte Carlo method for uncertainty calculation, with an application to the measurement of neutron ambient dose equivalent rate.* Radiat Prot Dosimetry (2006) 121 (1): 12-23



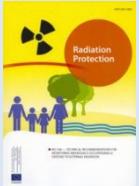
Evaluating uncertainty

IEC TR 62461 Table B1

Table B.1 – Example of an uncertainty budget for a photon dose measurement with a passive dosimetry system according to IEC 62387-1:2007 and low level of consideration of the workplace conditions, see text for details

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x ; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N_0	1,00	$0.05\sqrt{6} = 0,020\,4$	Triangular; $x = 1,0; a = 0,05$	10 mSv	0,20 mSv
K_n	1,00	$0.10\sqrt{3} = 0,057\,7$	Rectangular; $x = 1,0; a = 0,1$	10 mSv	0,58 mSv
$K_{E,\square\varphi}$	1,00	$0,40/3 = 0,133$	Gaussian; $x = 1,0; a = 0,4$	10 mSv	1,33 mSv
K_{add}	1,00	0	Rectangular; $x = 1,0; a = 0,0$	10 mSv	0,0 mSv
K_{temp}	1,00	$0,20/3 = 0,066\,7$	Gaussian; $x = 1,0; a = 0,20$	10 mSv	0,67 mSv
K_{light}	1,00	$0,1/3 = 0,033\,3$	Gaussian; $x = 1,0; a = 0,10$	10 mSv	0,33 mSv
K_{bup}	1,00	$0,1/3 = 0,033\,3$	Gaussian; $x = 1,0; a = 0,10$	10 mSv	0,33 mSv
K_{stab}	1,00	$0,1/3 = 0,033\,3$	Gaussian; $x = 1,0; a = 0,10$	10 mSv	0,33 mSv
K_{tempR}	1,00	$0,1/3 = 0,033\,3$	Gaussian; $x = 1,0; a = 0,10$	10 mSv	0,33 mSv
K_{lightR}	1,00	$0,1/3 = 0,033\,3$	Gaussian; $x = 1,0; a = 0,10$	10 mSv	0,33 mSv
K_{pow}	1,00	$0,1/3 = 0,033\,3$	Gaussian; $x = 1,0; a = 0,10$	10 mSv	0,33 mSv
G	10 mSv	$0,05 \times 10 \text{ mSv} = 0,50 \text{ mSv}$	Gaussian with one reading; $x = 10,0 \text{ mSv}; a = 1,50 \text{ mSv}$	10 mSv	0,33 mSv
$D_{\text{EMC},1}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$D_{\text{EMC},2}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$D_{\text{EMC},3}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$D_{\text{EMC},4}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$D_{\text{EMC},5}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$D_{\text{EMC},6}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$D_{\text{EMC},7}$	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
D_{dipo}	0 mSv	$0,7 \times 0,1 \text{ mSv}/3 = 0,023 \text{ mSv}$	Gaussian; $x = 0,0 \text{ mSv}; a = 0,07 \text{ mSv}$	-1,00	0,023 mSv
$H_p(10)$	10,0 mSv	1,9 mSv (19 %)	(Analytical method)		

***) Taken with permission from TR 62461 IEC:2014-08 45B final draft.**
Not to be used for reference purposes.



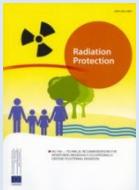
Evaluating uncertainty

— EURADOS →

The law of propagation of uncertainty, LPU, is based on the Taylor series development.

The measurand is for a value of the input quantity that slightly varies from the true value, approximated by a first degree Taylor series truncated after the 1st degree.

See NPL document ***dem_es11.pdf*** Chapter 6



Evaluating uncertainty

— EURADOS →

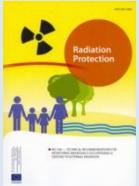
True value Small deviation due to uncertainty

$$\begin{aligned}y &= f(x) \\y_\Delta &= f(x + \Delta) \\&= f(x) + \frac{\partial f(x)}{\partial x} \Delta\end{aligned}$$

Negligible

$$+ \frac{\frac{\partial^2 f(x)}{\partial x^2}}{2!} \Delta^2 + \dots$$

Sensitivity coefficient



Evaluating uncertainty

— EURADOS →

The variance of the joint distribution of a series of normal distributions is the sum of the variances of the individual distributions.

Adding the squares of the product of sensitivity coefficient and standard uncertainty gives the variance in the measurand.

$$v_x = \left(\frac{\partial f(x)}{\partial x} u_x \right)^2$$

Assumptions:

The higher degree terms of the Taylor series can be neglected (~LPU)

The distribution of the measurand is approximately normal (~ CLT)