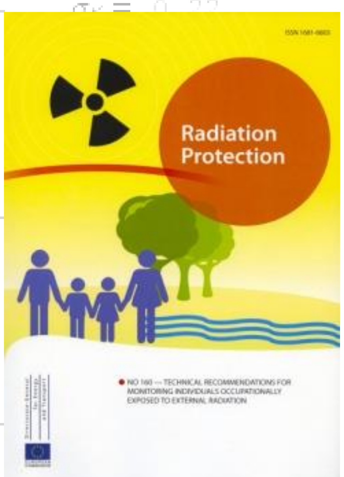
EURADOS Training course

Evaluating uncertainty

RP160 Chapter 5



Part 2

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Eurados Training Course
Lisbon, Portugal, 18-22 May 2015

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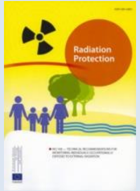

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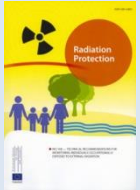
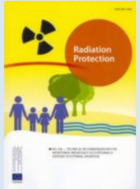
Uncertainty in measurement

II. The formulation stage and the GUM framework continued
III. The calculation stage and decision threshold and detection limit



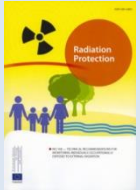
Uncertainty in measurement

- Quantitative measure for ***measurement quality***
- Uncertainty is an ***a-priory*** property of the system (as opposed to error in a measurement)
- Uncertainty evaluation requires to analyze the components of the measurement system



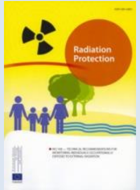
A few more terms

- **Standard uncertainty**: the standard deviation of the probability density function of the measurand
- **95% coverage interval**: the interval around the measured value that with 95% probability contains the true value



A few more terms

- **Decision threshold:** The value above which you can decide that a real dose above background level is observed
- **Detection limit:** The smallest true value of the dose that can be detected with the dosemeter



Stage 1.3 The measurement equation

In mathematical terms:

A function of the input quantities.

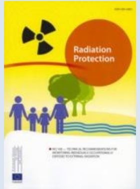
$$x_1, x_2, x_3, \ldots, x_N = X$$

$$y = f(X)$$

In practical terms:

A number of formulas used to calculate the dose.

Usually implemented in software.



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Stage 2 The calculation stage

Input quantities:

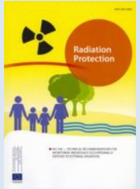
$$x_1,\, x_2,\, x_3,\, \ldots,\, x_N = X$$

Measurement model:

$$y = f(X)$$

Probability density functions for each x_i

$$g\left(x_i\right)$$



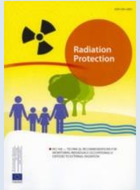
Probability density function of measurand

Probability density function g_y of the measurand

=

convolution integral of the joint distribution of $g_{\boldsymbol{X}}$

$$g_y(\eta) = \int \cdots \int_{-\infty}^{\infty} g_{\boldsymbol{X}}(\boldsymbol{\xi})\, \delta(\eta - f(\boldsymbol{\xi}))\, d\xi_N \ldots d\xi_1$$



Probability density function of measurand

$$g_y(\eta) = \int \cdots \int_{-\infty}^{\infty} g_{\boldsymbol{X}}(\boldsymbol{\xi})\, \delta(\eta - f(\boldsymbol{\xi}))\, d\xi_N \ldots d\xi_1$$

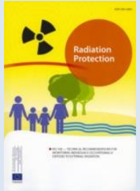
$$\downarrow$$

$$g_y(\eta) = h(X)$$

Does not exist (in general)

The two main solution routes

1. **The GUM framework method, LPU/CLT, JCGM 100:2008**
2. **The Monte Carlo Method, MCM, JCGM 101:2008**



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Solution routes

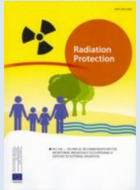
1. **Use of approximating functions**

   This is the GUM framework method of which the mathematics was developed by Gauß (end 18th century)

2. **Use numerical methods**

   This uses numerical integration based on Monte Carlo techniques

   (Laplace (1749–1827) reinvented 20th century)



Evaluating uncertainty

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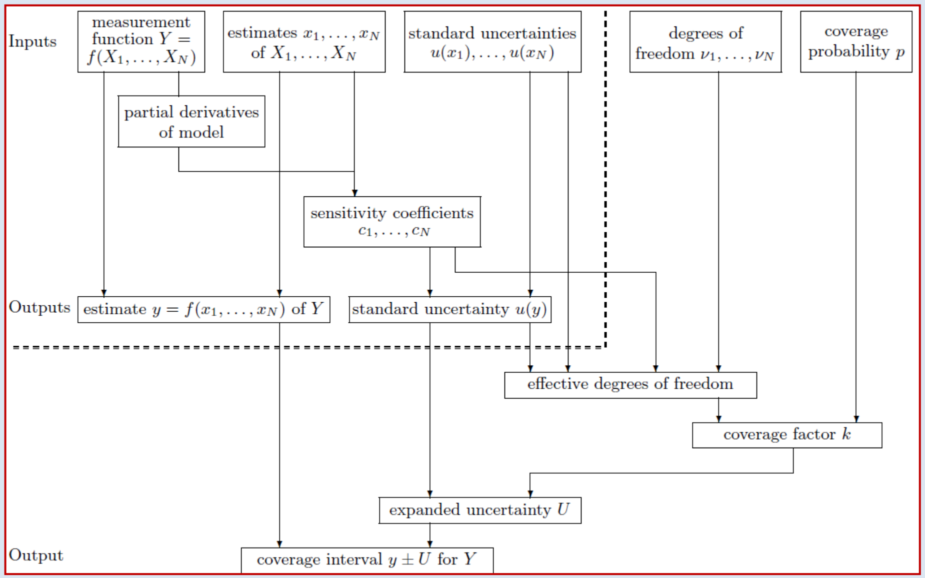
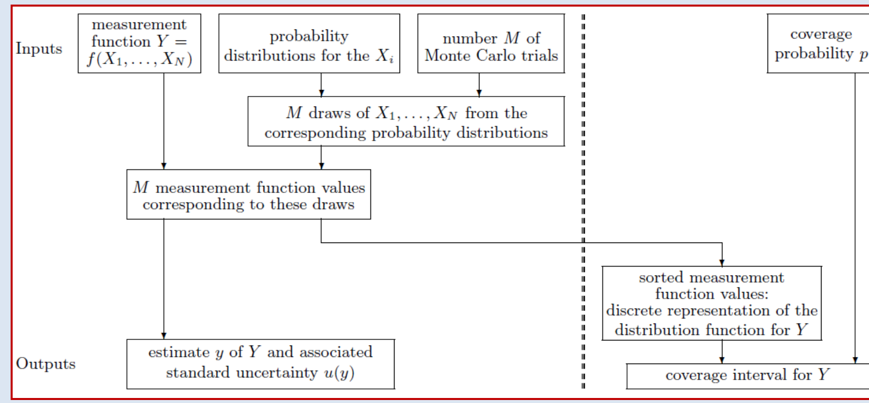
JCGM 104 figures 7 and 8

GUM Framework Method

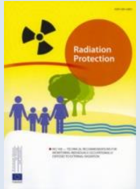


Monte Carlo Method



Full uncertainty evaluation until coverage interval

6 steps **4 steps**



Evaluating uncertainty

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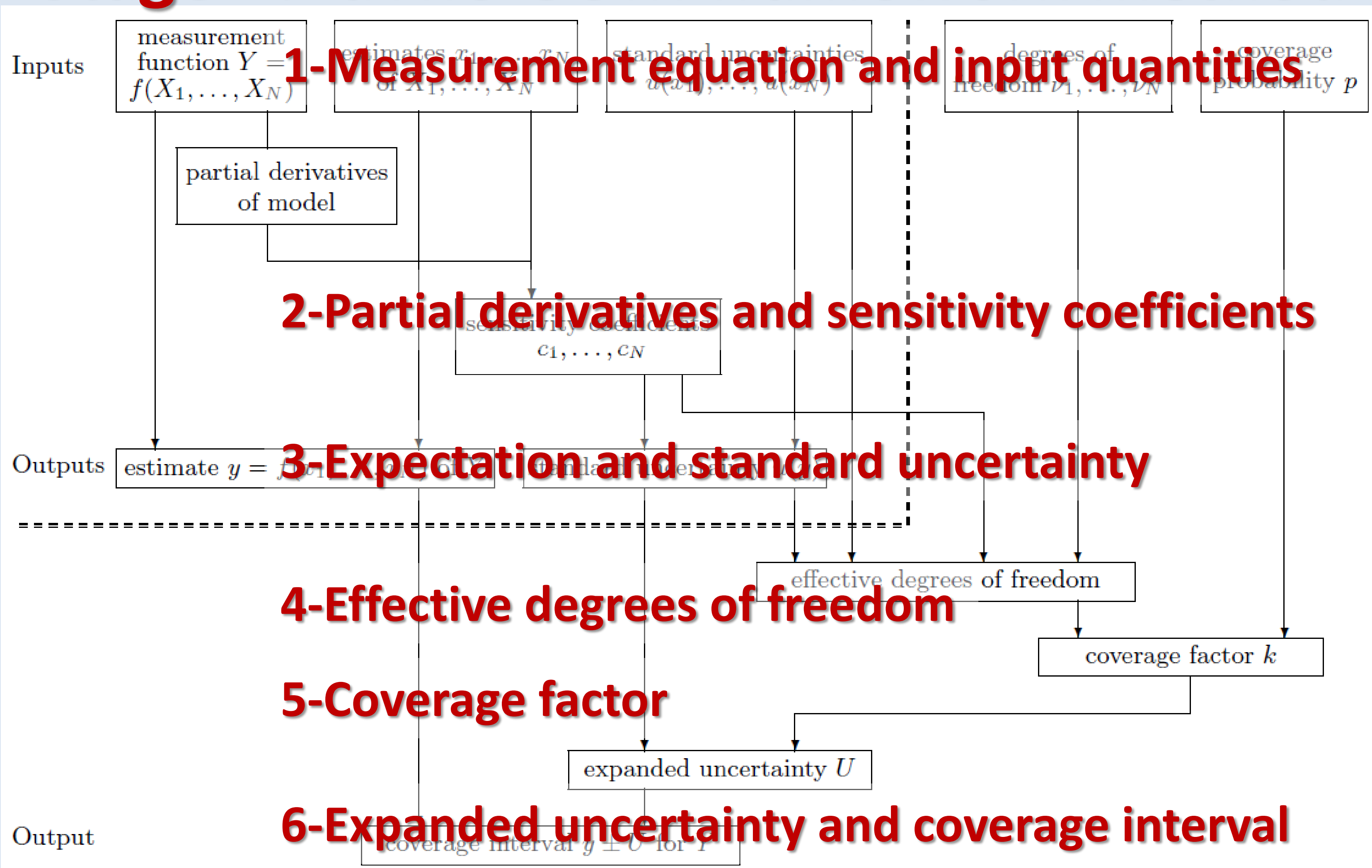
Stage 2.1aThe GUM framework method



1-Measurement equation and input quantities

2-Partial derivatives and sensitivity coefficients

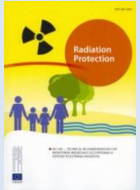
3-Expectation and standard uncertainty

4-Effective degrees of freedom

5-Coverage factor

6-Expanded uncertainty and coverage interval

JCGM 104:2009, fig 7, page 12



The GUM framework method

Based on the

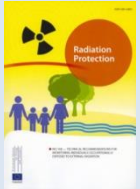
- **Law of Propagation of Uncertainty (LPU)**

$$u_y = \sqrt{\sum (c_i u_{x_i})^2}$$

u_y : standard uncertainty of y

c_i : sensitivity coefficient for x_i

- **The Central Limit Theorem (CLT)**
- **The resulting distribution g_y must be normal (more or less)**



The GUM framework method

1) The input quantities x_i and the measurement equation

$$x_1, x_2, \ldots\ldots, x_N = X$$

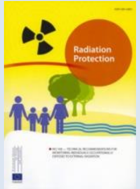
$$y = f\,(X)$$

$$\text{measurand} = \text{function of input quantities}$$

2) The sensitivity coefficients c_i

$$c_i = \frac{\partial y}{\partial x_i}$$

$$\text{sensitivity coeff.} = \text{partial derivative}$$



Evaluating uncertainty

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1 $$\text{measurand} = f\,(\text{input quantities})$$

$$y = f\,(x_1, x_2, ...x_N) = f\,(X)$$

2 $$\text{sensitivity coeff.} = \text{partial deriv.}$$

$$c_i = \frac{\partial f(X)}{\partial x_i}$$

3 $$\mathbf{LPU}: \boldsymbol{u_y} = \sqrt{\sum (\boldsymbol{c_i u_{x_i}})^2}$$

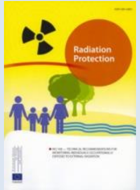
5 $$\text{expanded uncertainty}$$

$$U = k u_y$$

$$k\ : \text{coverage factor}$$

6 $$\text{coverage interval}$$

$$\hat{y} - U \leq y \leq \hat{y} + U$$



Coverage factor

Calculating the coverage coefficient involves

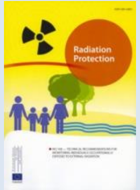
- The effective degrees of freedom η_i
- The Welch-Satterthwaite formula

  (GUM appendix G.4)

For most practical purposes:

$$k = 2$$

Will give the ≈95% coverage interval



The GUM framework method

LPU is an approximation

$$y = f\left(x_1, x_2, x_3\right) = x_1 x_2 + x_3$$

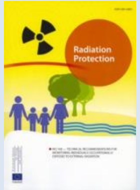
$$c_1 = x_2,\ c_2 = x_1,\ c_3 = 1$$

$$u_{y,\mathrm{LPU}} = \sqrt{\left(c_1 u_1\right)^2 + \left(c_2 u_2\right)^2 + u_3^2}$$

$$\gamma_{y,\mathrm{LPU}} = 0.0$$

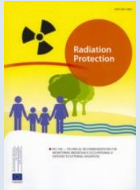
$$u_{\mathrm{y}} = \sqrt{\left(c_1 u_1\right)^2 + \left(c_2 u_2\right)^2 + u_3^2 + u_1^2 u_2^2}$$

$$\gamma_y = 6\, x_1 x_2\, u_1^2 u_2^2 \,/\, u_y^{2/3}$$



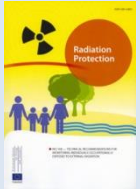
The application

First stage
The formulation stage



Components of a dosimetry system

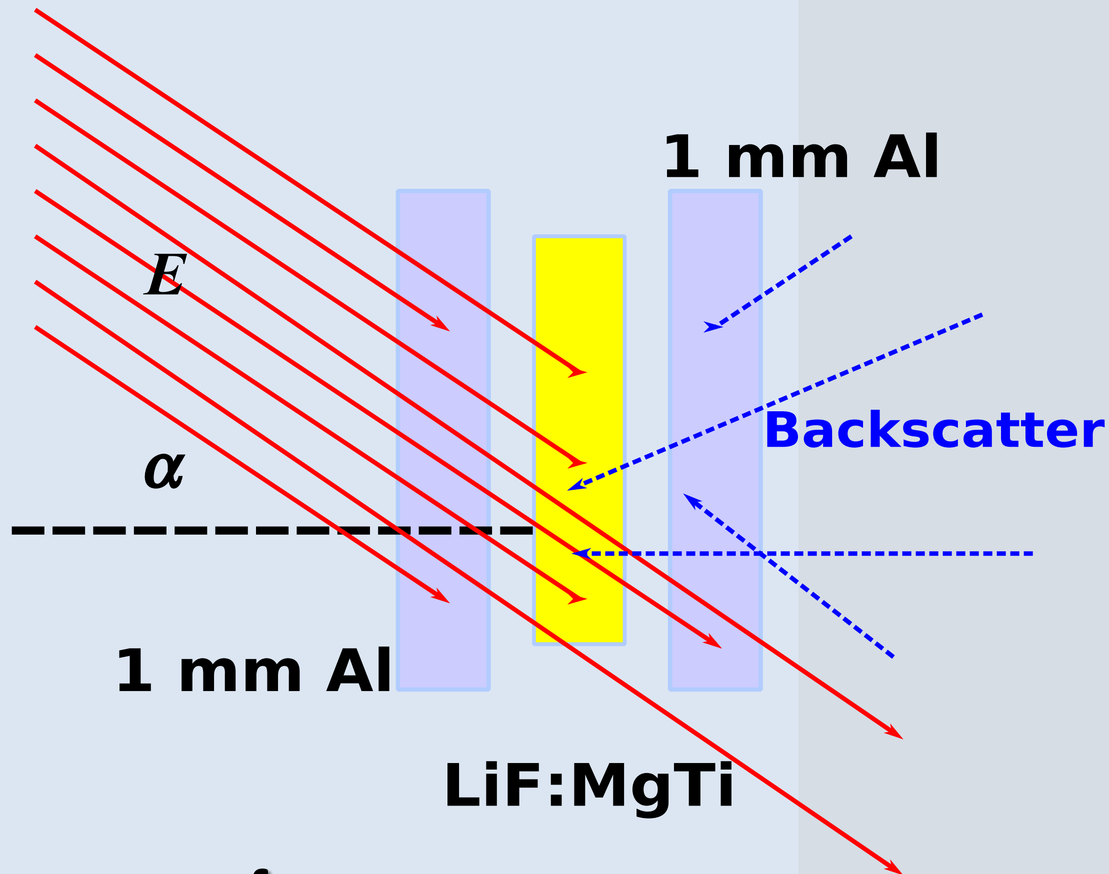
- Calibration source traceable to the national standard,
- Evaluation system (reader, developer, ..)
- Reference dosemeters,
- Custom dosemeters,
- Computers with software



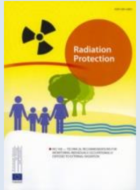
Components of a dosemeter

A very much simplified TL-dosemeter

- Single standard LiF:Mg,Ti detector
- 1 mm Al filter in front and behind

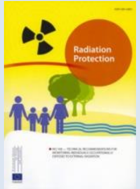


Type test data for this dosemeter from:
Radiat. Prot. Dosim. 54(3/4) 273-277 (1994)



Components of a measurement model

- **Sensitivity of the custom dosemeter relative to the reference dosemeter (element correction coefficient, ECC),**
- **Angle and energy dependency of response**
- **Sensitivity of the reader determined with reference dosemeter [signal dose^{-1}],**
- **Background dose,**
- **Other (environmental) influences**



The measurement model

$$x = f_{\mathrm{R}} f_{\mathrm{Det}} (f_{E,\alpha} H_{\mathrm{True}} + f_{\mathrm{Bg}} h_{\mathrm{Bg}}) + z$$

x : the signal [counts]

f_{R} : the reader sensitivity [counts μSv^{-1}]

f_{Det} : the relativedetector sensitivity (ecc) [1]

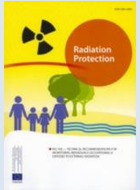
$f_{E,\alpha}$: the angle and energy dependent response, [1]

H_{True} : the true dose [μSv]

f_{Bg} : the response for background radiation, [1]

h_{Bg} : the background dose [μSv]

z : the blank signal [counts]



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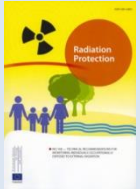
The measurement model

$$x = f_{\mathrm{R}} f_{\mathrm{Det}} (f_{E,\alpha} H_{\mathrm{True}} + f_{\mathrm{Bg}} h_{\mathrm{Bg}}) + z$$

$$\mathrm{signal} = \frac{\mathrm{ref}_{\mathrm{Cs},0}}{\mathrm{dose}} \frac{\mathrm{custom}}{\mathrm{ref}_{\mathrm{Cs},0}} \left\{ \frac{\mathrm{ref}_{E,\alpha}}{\mathrm{ref}_{\mathrm{Cs},0}} \mathrm{dose} + \frac{\mathrm{ref}_{E_{\mathrm{Bg}},0}}{\mathrm{ref}_{\mathrm{Cs},0}} \mathrm{dose} \right\} + \mathrm{signal}$$



Components of a measurement model

Solving for an unknown dose

$$y = \frac{x - z}{f_{\mathrm{R}} f_{\mathrm{Det}} f_{E,\alpha}} - \frac{f_{\mathrm{Bg}}}{f_{E,\alpha}} h_{\mathrm{Bg}}$$

or using correction coefficients $k_i = \frac{1}{f_i}$

$$y = k_{\mathrm{R}} k_{\mathrm{Det}} k_{E,\alpha}(x - z) - \frac{k_{\mathrm{E},\alpha}}{k_{Bg}} h_{\mathrm{Bg}}$$

International Electrotechnical Commission **IEC TR 62461**

Assigning distributions

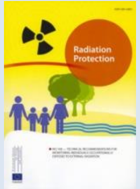
	Type	Distribution
x	A	normal
$f_{\mathrm{R}}, k_{\mathrm{R}}$	A	normal
$f_{\mathrm{Det}}, k_{\mathrm{Det}}$	B	normal
$f_{E,\alpha}, k_{E,\alpha}$	B	normal
$f_{\mathrm{Bg}}, k_{\mathrm{Bg}}$	B	normal
h_{Bg}	A	normal
z	A	normal

IEC International Electrotechnical Commission **IEC TR 62461 table B.1**

Uncertainty in measurement

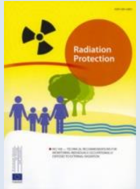
Think of all input quantities and the measurand as being a sample from a probability distribution

Doing measurements is obtaining random numbers from a distribution



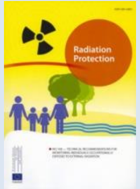
Second stage

The calculation stage
An example



Evaluating uncertainty

$$
\begin{aligned}
y &= k_{\mathrm{R}} k_{\mathrm{Det}} k_{E,\alpha}(x - z) - \frac{k_{\mathrm{E},\alpha}}{k_{Bg}} h_{\mathrm{Bg}} \\
\\
c_{k_R} &= k_{\mathrm{Det}} k_{E,\alpha}(x - z) \\
c_{k_{Det}} &= k_{\mathrm{R}} k_{E,\alpha}(x - z) \\
c_{k_{E,\alpha}} &= k_{\mathrm{R}} k_{\mathrm{Det}}(x - z) - \frac{1}{k_{Bg}} h_{Bg} \\
c_x &= k_R k_{Det} k_{E,\alpha} \\
c_z &= -k_R k_{Det} k_{E,\alpha} \\
c_{k_{Bg}} &= -\frac{k_{E,\alpha}}{k_{Bg}^2} h_{Bg} \\
c_{h_{Bg}} &= \frac{k_{E,\alpha}}{k_{Bg}}
\end{aligned}
$$



Evaluating uncertainty

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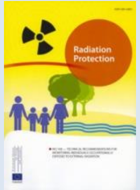
LiF:Mg,Ti + 1mm Al

	0	20	40	60
17.4	0.63	0.55	0.44	0.20
17.4	0.61	0.53	0.43	0.22
17.4	0.62	0.52	0.51	0.30
23.1	1.15	1.09	1.00	0.76
23.1	1.09	1.10	0.97	0.75
23.1	[illegible]	1.11	1.02	0.75
[illegible]	[illegible]	1.18	1.12	0.94
[illegible]	1.20	1.36	1.16	0.89
25.2	1.18	1.21	1.12	0.91
30.9	1.34	1.31	1.36	1.21
30.9	1.32	1.38	1.43	1.27
30.9	1.32	1.35	1.29	1.19
33	1.35	1.37	1.33	1.40
33	1.36	1.32	1.37	1.40
33	1.45	1.39	1.42	1.43
48	1.36	1.37	1.43	1.38
48	1.39	1.39	1.48	1.49
48	1.40	1.38	1.43	1.47
65	1.23	1.26	1.35	1.37
65	1.27	1.26	1.27	1.32
65	1.39	1.26	1.34	1.39
83	1.28	1.19	1.23	1.29
83	1.31	1.23	1.29	1.29
83	1.24	1.25	1.25	1.23
100	1.21	1.18	1.16	1.23
100	1.17	1.13	1.22	1.25
100	1.18	1.20	1.19	1.23
118	1.17	1.16	1.17	1.26
118	1.21	1.17	1.21	1.22
118	1.16	1.14	1.18	1.20
161	1.08	1.18	1.13	1.18
161	1.16	1.16	1.16	1.20
161	1.15	1.13	1.16	1.15
205	1.07	1.17	1.13	1.16
205	1.12	1.12	1.13	1.17
205	1.13	1.14	1.14	1.17
248	1.16	1.12	1.08	1.25
248	1.13	1.09	1.04	1.16
248	1.13	1.15	1.14	1.18
662	1.16	1.11	1.09	1.07
662	1.03	1.09	1.12	1.09
662	1.11	1.10	1.10	1.12
1250	0.98	0.99	1.03	1.01
1250	1.02	1.03	1.00	1.01
1250	1.00	0.98	0.98	1.03

Data triplet

Energy	Angle			
keV	0	20	40	60
161	1.08	1.18	1.13	1.18
161	1.16	1.16	1.16	1.20
161	1.15	1.13	1.16	1.15
205	1.07	1.17	1.13	1.16
205	1.12	1.12	1.13	1.17
205	1.13	1.14	1.14	1.17
248	1.16	1.12	1.08	1.25
248	1.13	1.09	1.04	1.16
248	1.13	1.15	1.14	1.18
662	1.16	1.11	1.09	1.07
662	1.03	1.09	1.12	1.09
662	1.11	1.10	1.10	1.12

15 photon energies,
4 angles of incidence
In triplicate (180 data)



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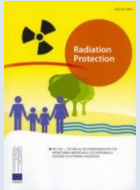
Numerical values

	expext.	range	u_i
x	$1065\,100$	–	$10\,651$
k_{R}	$10^{-3}\mu\mathrm{Sv}$	–	$0.05\,10^{-3}$
k_{Det}	1.00	$0.85 - 1.15$	0.05
$k_{E,\alpha}$	1.00	$0.55 - 1.45$	0.15
k_{Bg}	1.00	$0.85 - 1.15$	0.05
h_{Bg}	$65\,\mu\mathrm{Sv}$	–	6.5
z	100	–	5



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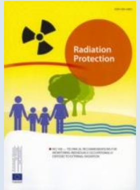
Homework:

Use the equations in earlier sheets to calculate the

1. **Dose**
2. **Standard uncertainty**
3. **Approximate 95% coverage interval**



Evaluating uncertainty

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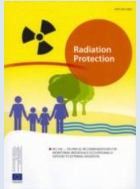
Now forget the

- **LPU**
- **CLT**
- **Partial derivatives**
- **Welch-Satterthwaite**
- **Coverage factor**

Solve

$$g_y(\eta) = \int \cdots \int_{-\infty}^{\infty} g_{\boldsymbol{X}}(\boldsymbol{\xi})\, \delta(\eta - f(\boldsymbol{\xi}))\, d\xi_N \ldots d\xi_1$$

By Monte Carlo integration



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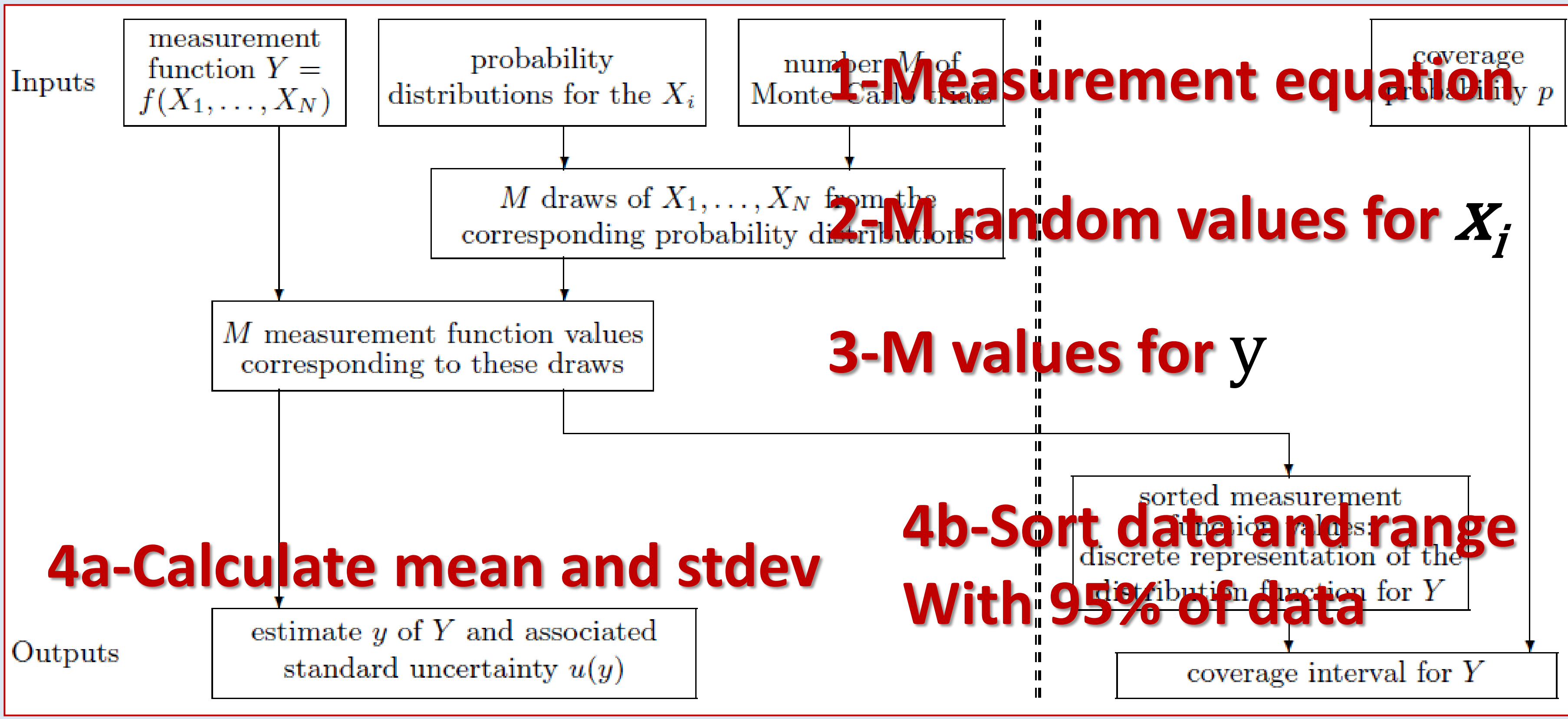
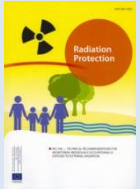
Stage 2.1b The Monte Carlo method



JCGM 104:2009, fig 8, page 13



Evaluating uncertainty

EURADOS

A 23 line Python program doing complete MCM

```
import numpy.random as random
from scipy import stats as stats
#Define variables
M = 1000000  # number of Monte Carlo samples
X = {'x': 1065100, 'k_R': 1e-3, 'k_Det': 1.0, 'k_Ea': 1.0,
     'k_Bg': 1.0, 'h_Bg': 65.0, 'z': 100.0}
UX = {'x': X['x'] * 0.01, 'k_R': X['k_R'] * 0.05, 'k_Det': 0.05,
      'k_Ea': 0.15, 'k_Bg': 0.05, 'h_Bg': 6.5, 'z': 5.0}
#Generate M random numbers for input quantities
Xr = {}  # will contain the random values for X
for k in X:
   Xr[k] = random.normal(X[k], UX[k], M)
#calculate M values for measurand
y = Xr['k_R'] * Xr['k_Det'] * Xr['k_Ea'] * (Xr['x'] - Xr['z']) -\
   Xr['h_Bg'] * Xr['k_Ea'] / Xr['k_Bg']

y.sort()
mean = y.mean()
stdev = y.std()
skew = stats.skew(y)
print('Mean:     %.2f' % mean)
print('Stdev:    %.3f' % stdev)
print('Skew:     %.3f' % skew)
print('Coverage: %.2f - %.2f' % (y[p], y[M-p]))
```

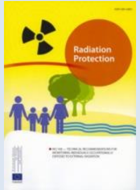
Output:

Mean: 999.71

Stdev: 168.56

Skew: 0.176

Coverage: 683.34 - 1345.33



EURADOS

Stage 2.1b The Monte Carlo method

On the practicum:
Doing MCM with a spread-sheet

Decision and detection thresholds