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# Universal curve of epithermal neutron resonance self-shielding factors in foils, wires and spheres

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**Abstract**

The presence of a nuclide sample in an epithermal neutron field of a nuclear reactor creates a perturbation of the local neutron flux. This effect can be very important, especially if the nuclide cross-section exhibits a prominent resonance peak. To take into account the effect of the neutron flux perturbation in the sample activation, a resonance neutron self-shielding factor ($G_{res}$) must be considered. This factor depends on the geometry and dimension of the sample, as well as on the physical and nuclear properties of the nuclide. On the basis of a dimensionless variable which includes the relevant characteristics of the sample, an universal curve is proposed, which enables the determination of the factor $G_{res}$ for isolated resonances of any nuclide and samples of various geometries (foils, wires and spheres). The proposed universal curve is in very good agreement with experimental and calculated values obtained from the literature.


*Keywords:* Epithermal neutrons; Resonance self-shielding factor; MCNP code

## 1. Introduction

The irradiation of a sample in an epithermal neutron field of a nuclear reactor is affected by the perturbation of the neutron flux produced by the sample. The interpretation of the sample activation data requires the application of a resonance neutron self-shielding factor, $G_{res}$. The value of $G_{res}$ depends on the physical and nuclear properties of the nuclide as well as on the sample geometry (foil, wire or sphere) and the typical dimension (radius for wires and spheres, thickness for foils).

Some experimental (Eastwood and Werner, 1962; Axton, 1963; Baumann, 1963; Brose, 1964; McGarry, 1964; De Corte, 1987; Freitas, 1993) and theoretical studies (Selander, 1960; Baumann, 1963; Brose, 1964; Yamamoto and Yamamoto, 1965; Jefferies et al., 1983; Kumpf, 1986; Lopes, 1991) have been carried out to measure or to calculate the resonance self-shielding effect induced by samples (wires or foils) of different nuclides. The results of these studies were presented as tables or graphics of $G_{res}$ for a given nuclide and geometry as a function of the typical dimension.

A systematic study of $G_{res}$ for different nuclides was performed for wires (Gonçalves et al., 2001) and for foils (Gonçalves et al., 2002). These studies showed that a dimensionless variable can be introduced, which converts, for each geometry, the dependence of the resonance self-shielding factor on physical and nuclear parameters into an universal curve valid for all nuclides.

In the present work, the study is extended to spherical samples and an universal curve, valid for all nuclides and the studied geometries, is deduced.

## 2. Neutron activation

According to the Westcott formalism (Westcott, 1960), the neutron activation, $A^*$, induced in a sample by radiative neutron capture is given by

$$A^* = N\left[g\sigma_0\Phi_0G_{th} + \frac{2}{\sqrt{\pi}}\Phi_{epi}G_{res}(I_\gamma - 0.45\sigma_0)\right] \times (1 - e^{-\lambda t}), \quad (1)$$

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where $N$ is the number of atoms ${}^{A}X$ present in the sample, $\sigma_0$ is the thermal neutron cross-section for the $(n,\gamma)$ reaction ($v_0 = 2200\,\mathrm{m\,s^{-1}}$), $g$ is a parameter representing the deviation of the $(n,\gamma)$ cross-section in the thermal region from the $1/v$ law, $I_\gamma$ is the radiative neutron capture resonance integral, $\Phi_0$ is the thermal neutron flux, $\Phi_{epi}$ is the epithermal neutron flux, $\lambda$ is the decay constant of the nuclide ${}^{A+1}X$, $t$ is the irradiation time, $G_{th}$ is the thermal neutron self-shielding factor, and $G_{res}$ is the resonance neutron self-shielding factor. The focus of this work is on a method to calculate $G_{res}$.

## 3. Methodology of calculation

In the epithermal region, the neutron cross-section exhibits, normally, several well defined and isolated resonances. It is nearly always enough to calculate $G_{res}$ for the most important resonance, as in the case of ${}^{197}$Au for the resonance energy $E_{res} = 4.9\,\mathrm{eV}$.

The following nuclides were used in the calculation of $G_{res}$: ${}^{55}$Mn, ${}^{59}$Co, ${}^{63}$Cu, ${}^{115}$In, ${}^{185}$Re and ${}^{197}$Au. Table 1 shows their physical and nuclear parameters, where $A$ is the atomic mass, $\rho$ is the density, $\theta$ is the natural abundance, $\Gamma_\gamma$ and $\Gamma_n$ are, respectively, the resonance widths for $(n,\gamma)$ and $(n,n)$ reactions, and $\Gamma = \Gamma_\gamma + \Gamma_n$. As can be seen, these parameters are quite different:

(a) The resonance energy varies from 1.46 eV (${}^{115}$In) to 579 eV (${}^{63}$Cu);
(b) The cross-section at the resonance peak varies from 918 b (${}^{63}$Cu) to 31150 b (${}^{115}$In);
(c) The ${}^{115}$In and ${}^{197}$Au resonances are dominated by radiative capture ($\Gamma_\gamma/\Gamma \geqslant 0.89$); they are good absorbers;
(d) The ${}^{55}$Mn and ${}^{59}$Co resonances are, predominantly, due to scattering ($\Gamma_n/\Gamma \geqslant 0.91$); they are good scatterers;
(e) The contributions of scattering and radiative capture in the case of ${}^{63}$Cu are practically the same.

For a given nuclide, geometry and sample dimension, $G_{res}$ was calculated by using the MCNP code (Briesmeister, 2000). The resonance self-shielding factor is calculated as the ratio between the reaction rates per atom in the real sample and that in a similar and infinitely diluted sample. Thus, the expression for $G_{res}$ is obtained from:

$$G_{res}(x) = \frac{\int_{E_1}^{E_2} \Phi(E)\sigma_{n\gamma}(E)\,\mathrm{d}E}{\int_{E_1}^{E_2} \Phi_0(E)\sigma_{n\gamma}(E)\,\mathrm{d}E}, \tag{2}$$

where $x$ is the sample dimension, $\Phi_0(E) \propto E^{-1}$ is the non-perturbed epithermal neutron flux per unit energy interval (inside the infinitely diluted sample), $\Phi(E)$ is the perturbed epithermal neutron flux inside the real sample, $\sigma_{n\gamma}(E)$ designates the $(n,\gamma)$ cross-section, and $E_1$ and $E_2$ are, respectively, the lower and the upper limits around the resonance energy $E_{res}$. The total neutron cross-section has been adopted in the calculation of the perturbed neutron flux, which takes into account the neutron scattering in the sample. In all calculations the density for infinite dilution was assumed to be $\rho = 10^{-6}\,\rho_0$, $\rho_0$ representing the density of the real sample (Gonçalves et al., 2001).

In recent papers, using wires and foils Gonçalves et al. have shown that it is possible to introduce a dimensionless variable, $z$, which converts the dependence of $G_{res}$ on the dimension and physical and nuclear parameters into an unique curve (Gonçalves et al., 2001, 2002). This variable is defined as

$$z = \Sigma_{tot}(E_{res})x(\Gamma_\gamma/\Gamma)^{1/2}, \tag{3}$$

where $\Sigma_{tot}(E_{res})$ is the macroscopic cross-section at the resonance peak, $\Gamma_\gamma$ is the resonance width for the $(n,\gamma)$ reaction, $\Gamma$ is the total resonance width ($\Gamma = \Gamma_\gamma + \Gamma_n$) and $x$ is the typical sample dimension—$x = t$ (thickness) for foils, and $x = R$ (radius) for wires.

A re-analysis of the previous results shows that a sigmoid is the best curve to be fitted to the calculated values. The expression of this curve is

$$G_{res} = \frac{A_1 - A_2}{1 + (z/z_0)^p} + A_2, \tag{4}$$

where $A_1$, $A_2$, $z_0$ and $p$ are the curve parameters, to be adjusted to the calculated values. Note that

(a) $A_1$ is the limit of $G_{res}$ as $z$ tends to zero;
(b) $A_2$ is the limit of $G_{res}$ as $z$ tends to infinity;
(c) $z_0$ is the inflexion point $[G_{res}(z_0) = (A_1 + A_2)/2]$;
(d) $p$ is related with the gradient of the curve at $z = z_0$.

Table 1
Physical and nuclear parameters of the studied nuclides (Tuli, 2000; KAERI; LANL)

| Nuclide | $A$ (g mol$^{-1}$) | $\rho$ (g cm$^{-3}$) | $\theta$ (%) | $E_{res}$ (eV) | $\sigma_{tot}(E_{res})$ (b) | $\Gamma_\gamma$ (eV) | $\Gamma_n$ (eV) | $\Gamma = \Gamma_\gamma + \Gamma_n$ (eV) | $\Gamma_\gamma/\Gamma_n$ (%) |
|---|---|---|---|---|---|---|---|---|---|
| ${}^{197}$Au | 196.97 | 19.3 | 100 | 4.91 | 30 770 | 0.1225 | 0.0152 | 0.1377 | 89.0 |
| ${}^{59}$Co | 58.93 | 8.9 | 100 | 132 | 10 370 | 0.47 | 5.27 | 5.74 | 8.2 |
| ${}^{63}$Cu | 63.55 | 8.96 | 69.17 | 579 | 918 | 0.485 | 0.59 | 1.075 | 45.1 |
| ${}^{115}$In | 114.82 | 7.31 | 95.71 | 1.46 | 31 150 | 0.072 | 0.00304 | 0.07504 | 95.9 |
| ${}^{55}$Mn | 54.94 | 7.32 | 100 | 337 | 3290 | 0.31 | 21.99 | 22.30 | 1.4 |
| ${}^{185}$Re | 186.2 | 21.02 | 37.40 | 2.16 | 24 550 | 0.0549 | 0.00283 | 0.05773 | 95.1 |

## 4. Results and discussion

The resonance self-shielding factor for spheres with different radii is shown in Fig. 1 as a function of the dimensionless variable $z$. The sigmoid fits well to the calculated points; only some $^{63}$Cu values show a deviation from the curve. Figs. 2 and 3 represent the factor $G_{res}$ for wires and foils, as published before (Gonçalves et al., 2001, 2002). It is also evident that the sigmoid fits conveniently the calculated values. Table 2 shows the sigmoid parameters for the three cases.

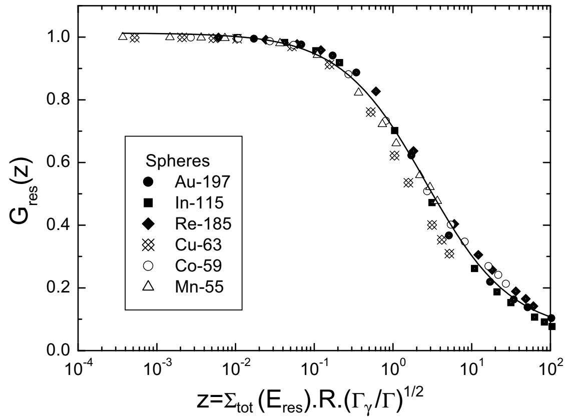


Fig. 1. Resonance self-shielding factor of samples in form of spheres as a function of $z = \Sigma_{tot}(E_{res})R(\Gamma_\gamma/\Gamma)^{1/2}$, where $R$ is the sphere radius.

The analysis of these results shows that, in fact, $A_1$, $A_2$ and $p$ are constants, and $z_0$ is variable. Assuming $A_1 = 1$ due to physical reasons and using the relationship

$$G_{res}(z_0) = \frac{A_1 + A_2}{2} = \frac{1.01 + 0.05}{2} = \frac{1 + A_2}{2}, \tag{5}$$

it follows that $A_2 = 0.06$. The parameter $z_0$ depends on the sample geometry. We can assume that $z_{0,sph}/z_0 = 1$ for spheres, 1.5 for foils and 2 for wires.

Using a new variable transformation

$$y = \frac{z_{0,sph}}{z_0} x \tag{6}$$

all the values of $G_{res}$ for wires and foils are translated upon the values of the spheres and an unique curve, the "universal curve", can fit all values, as is shown in Fig. 4. The parameters of the universal curve are shown in Table 3. Then the sigmoid

$$G_{res} = \frac{0.94}{1 + (z/2.70)^{0.82}} + 0.06 \tag{7}$$

can be used to evaluate $G_{res}$ for any new sample (any geometry and composition).

Fig. 5 shows the comparison of the universal curve with experimental values obtained by different authors (Eastwood and Werner, 1962; Axton, 1963; Baumann,

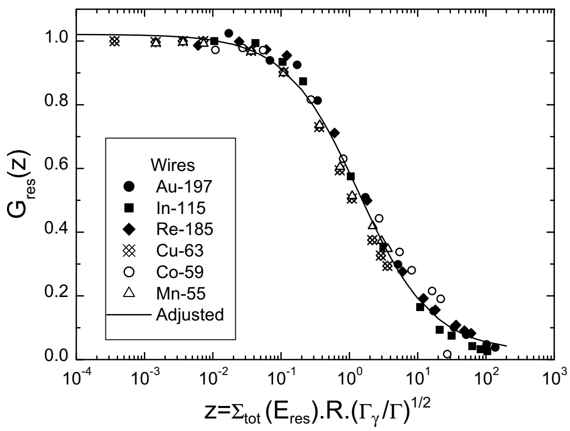


Fig. 2. Resonance self-shielding factor of samples in form of wires as a function of $z = \Sigma_{tot}(E_{res})R(\Gamma_\gamma/\Gamma)^{1/2}$, where $R$ is the wire radius.

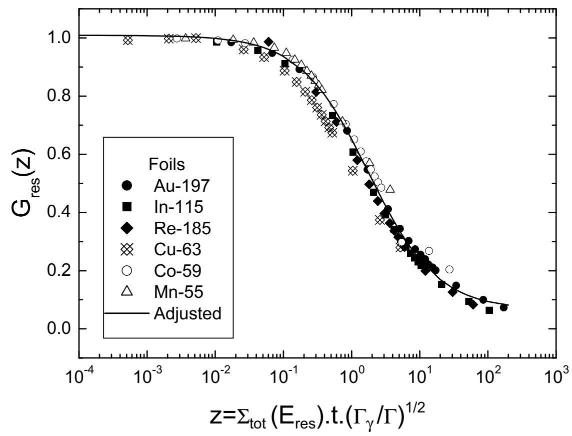


Fig. 3. Resonance self-shielding factor of samples in form of foils as a function of $z = \Sigma_{tot}(E_{res})t(\Gamma_\gamma/\Gamma)^{1/2}$, where $t$ is the foil thickness.

Table 2
Parameters of the sigmoid curves

| Geometry | Dimension | $A_1$ | $A_2$ | $z_0$ | $p$ | $z_{0,sph}/z_0$ |
|---|---|---|---|---|---|---|
| Wires | $x = R$ | $1.02 \pm 0.01$ | $0.025 \pm 0.015$ | $1.38 \pm 0.09$ | $0.81 \pm 0.04$ | $1.92 \pm 0.15$ |
| Foils | $x = t$ | $1.01 \pm 0.01$ | $0.066 \pm 0.013$ | $1.77 \pm 0.08$ | $0.86 \pm 0.03$ | $1.49 \pm 0.12$ |
| Spheres | $x = R$ | $1.01 \pm 0.01$ | $0.058 \pm 0.012$ | $2.64 \pm 0.13$ | $0.80 \pm 0.03$ | 1 |
| Mean value | | $1.01 \pm 0.02$ | $0.050 \pm 0.023$ | | $0.82 \pm 0.06$ | |

Table 3
Parameters of the universal curve

| Geometry | Dimension | $A_1$ | $A_2$ | $z_0$ | $P$ |
|---|---|---|---|---|---|
| Wires | $y = 2 \times R$ | $1.000 \pm 0.005$ | $0.060 \pm 0.011$ | $2.70 \pm 0.09$ | $0.82 \pm 0.02$ |
| Foils | $y = 1.5 \times t$ | | | | |
| Spheres | $y = R$ | | | | |

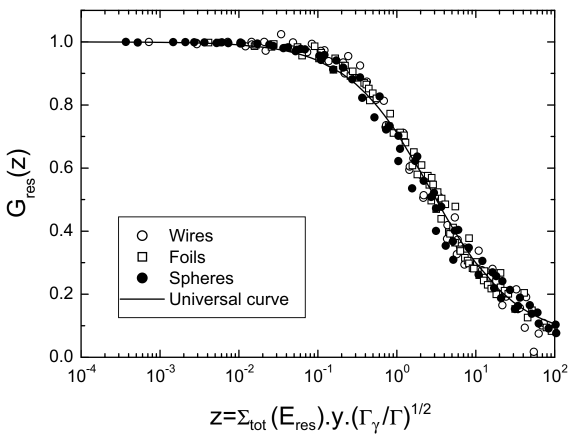


Fig. 4. Universal curve of the resonance self-shielding factor as a function of $z = \Sigma_{tot}(E_{res})y(\Gamma_\gamma/\Gamma)^{1/2}$, where $y = R$ for spheres, $y = 1.5t$ for foils and $y = 2R$ for wires.

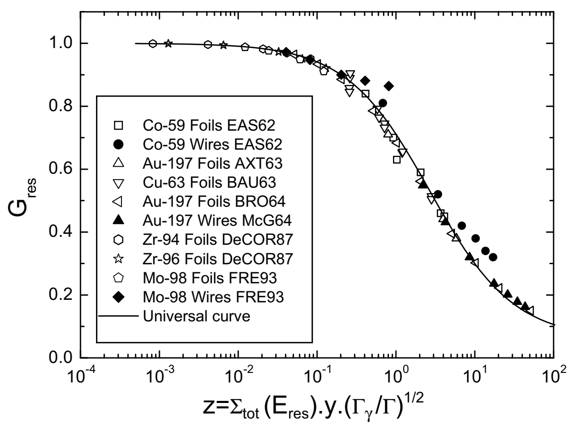


Fig. 5. Comparison of the universal curve with experimental self-shielding factors.

1963; Brose, 1964; McGarry, 1964; De Corte, 1987; Freitas, 1993). The experimental values include results of nuclides studied in the present work, as well as of others with very different physical and nuclear properties. The agreement is excellent. The average deviation of the experimental values relatively to the universal curve is equal to 4%.

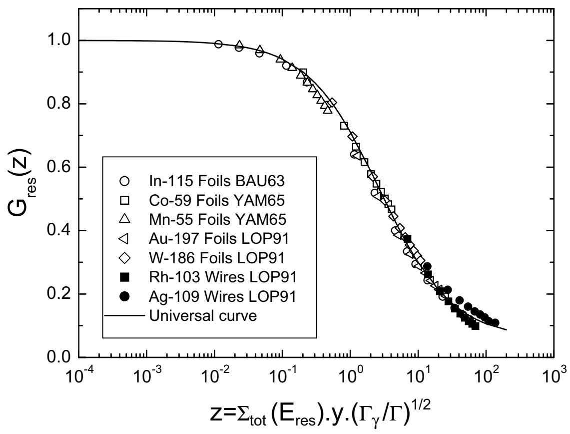


Fig. 6. Comparison of the universal curve with theoretical self-shielding factors.

Fig. 6 shows the comparison of the universal curve with theoretical values calculated by other authors (Baumann, 1963; Yamamoto and Yamamoto, 1965; Lopes, 1991). The agreement is also very good.

## 5. Conclusions

In this paper, it has been shown that a dimensionless variable can be introduced which converts the dependence of the resonance neutron self-shielding factor of wires, foils and spheres on physical and nuclear parameters in an universal curve valid for isolated resonances of any nuclide. The average deviation of experimental published values relatively to the universal curve is in the order of 4%.

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